# Thermodynamic

# 2026-1

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**NAUDY LOPEZ**

# $S_{4\_1\_}$ Ideal Gas Mixture

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## Recomendaciones generales:

- Leer antes de clase y entender los conceptos o preguntar si es necesario.
- Practicar bastante:
  - *Ejercicios de clase
  - *Asesorías
  - *Ejercicios del libro resueltos y los de final de capitulo
- Los quizzes virtuales  y gradescope úsenlos para aprender de forma honesta (se puede inferir).
- Tengan grupos de estudio y apóyense mutuamente.
- Crear sus propias notas estudio y formulario.
- Son 4 horas de clase en la semana (asistir y ser puntuales, perder alguna es atrasarse mucho)
- Cada hora de clase son 3 horas de estudio por fuera mínimo.
- En la ppt de la semana 1 están todos los links claves para el curso.

# Thermodynamics _ Evaluation

| COMPONENTE | % |
|---|---|
| PARCIAL | 20 |
| FINAL | 30 |
| C | 50 |
| APROBACIÓN: 11/20 | |

| | | |
|---|---|---|
| SEMANA 1 | TAREA VIRTUAL | SE CALIFICA AUTOMATICO NO NECESITA PROCEDIMIENTO |
| SEMANA 2 | TAREA VIRTUAL | |
| SEMANA 3 | TAREA VIRTUAL | . |
| **SEMANA 4** | **TAREA INDIVIDUAL 1** + EVALUACION_ AULA1 | TAREA INDIVIDUAL: Esta parte incluye subir un archivo con la solución<br>EVALUACION_ AULA: SEMANAS 1-2 |
| SEMANA 5 | TAREA VIRTUAL+ EVALUACION_ AULA2 | EVALUACION_ AULA: SEMANAS 1-3 |
| **SEMANA 6** | TAREA VIRTUAL**+TAREA INDIVIDUAL 2** | |
| SEMANA 7 | TAREA VIRTUAL+ EVALUACION_ AULA3 | EVALUACION_ AULA: SEMANAS 1-6 |
| **SEMANA 8** | **PARCIAL** | |
| SEMANA 9 | EVALUACION_ AULA4+**TAREA INDIVIDUAL 3 +Tarea grupal 1** | EVALUACION_ AULA: SEMANAS 1-8 |
| SEMANA 10 | TAREA VIRTUAL | |
| SEMANA 11 | TAREA VIRTUAL | |
| SEMANA 12 | TAREA VIRTUAL+ EVALUACION_ AULA5 | EVALUACION_ AULA: SEMANAS 1-11 |
| **SEMANA 13** | TAREA VIRTUAL**+TAREA INDIVIDUAL 4 + Tarea grupal 2** | |
| SEMANA 14 | TAREA VIRTUAL+EVALUACION_ AULA6 | EVALUACION_ AULA : SEMANAS 1-13 |
| SEMANA 15 | | |
| SEMANA 16 | **FINAL** | |

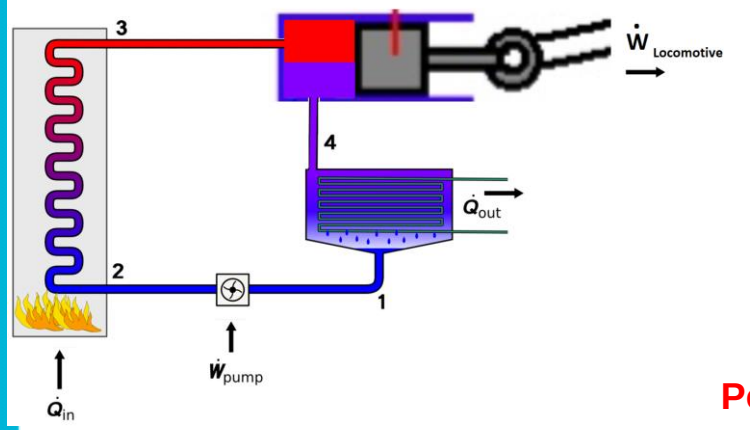
***LAS TAREAS VIRTUALES O LOS GRADESCOPE (TAREA INDIVIDAL) se abren lunes 2pm y se cierran sábado media noche. La tarea virtual se presenta hasta un máximo de 3 veces.**

EVALUACION_ AULA: es un quiz de 20 min aprox. en aula.

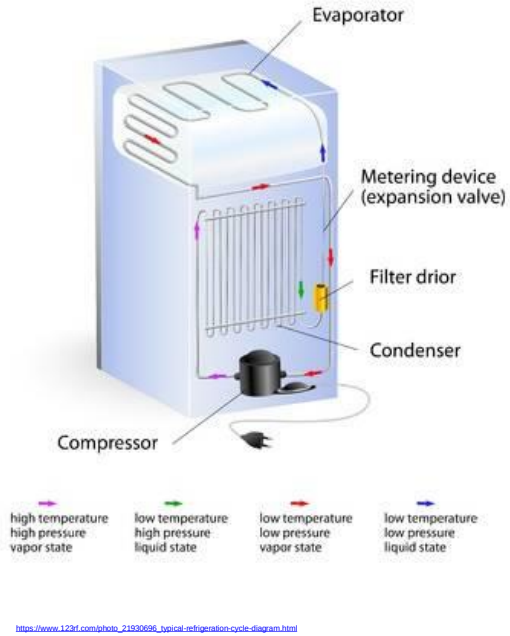
**Session 7-8**: Apply the ideal gas model for thermodynamic analysis of mixtures using Dalton and Amagat's law.

# Thermodynamics?

## Power cycle_ Rankine cycle (ideal)

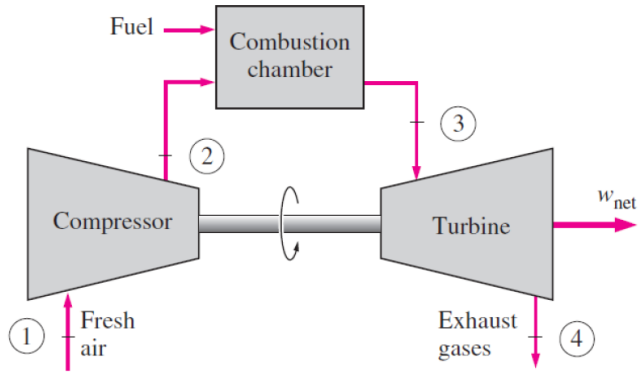



## Refrigeration cycle_ Vapor-Compression Refrigeration Cycle




https://www.123rf.com/photo_21930696_typical-refrigeration-cycle-diagram.html

## Power cycle_ Brayton cycle (ideal)

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1:57


https://www.youtube.com/watch?v=XDkqcTQTCr4

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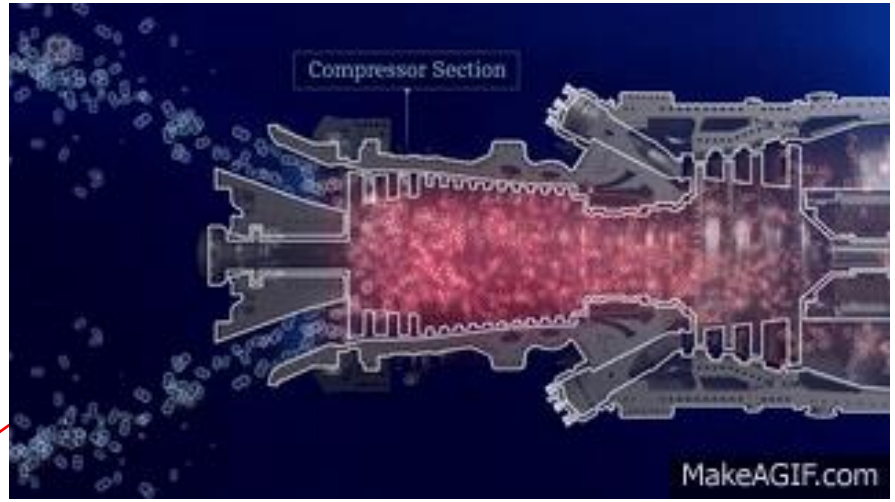
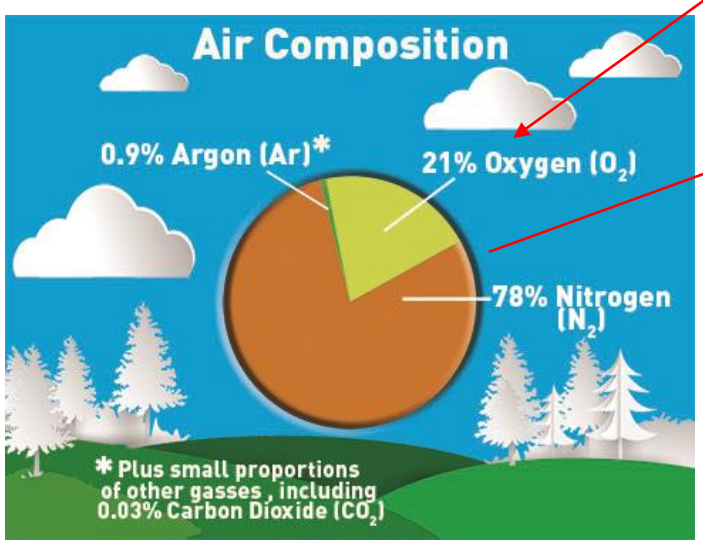
# Specific objectives

- Apply the rules for determining mixture properties (**u, h**) to ideal gas mixtures.
- Estimate **mass fractions, molar fractions, average molar mass.. $C_{vm}$..$C_{pm}$....** for ideal gases mixtures.



https://makeagif.com/i/bFyWSW



http://www.gettingthepicture.info/2/

**TABLE A-22**

**Ideal Gas Properties of Air**

*T*(K), *h* and *u*(kJ/kg), $s^\circ$ (kJ/kg · K)

| *T* | *h* | *u* | $s^\circ$ |
|---|---|---|---|
| 200 | 199.97 | 142.56 | 1.29559 |
| 210 | 209.97 | 149.69 | 1.34444 |
| 220 | 219.97 | 156.82 | 1.39105 |
| 230 | 230.02 | 164.00 | 1.43557 |
| 240 | 240.02 | 171.13 | 1.47824 |

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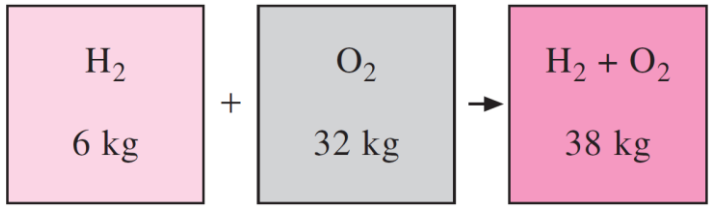
# COMPOSITION OF A GAS MIXTURE:

## MASS FRACTIONS




The mass of a mixture is equal to the sum of the masses of its components

$$m_m = \sum_{i=1}^{k} m_i$$

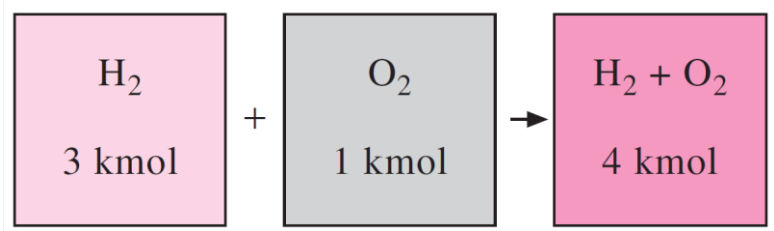
$$\text{mf}_i = \frac{m_i}{m_m}$$

$$\text{mf}_{O_2} = \frac{32 \text{ kg}}{38 \text{ kg}} = 0.84$$

$$\text{mf}_{H_2} = \frac{6 \text{ kg}}{38 \text{ kg}} = 0.16$$

$$\sum_{i=1}^{k} \text{mf}_i = 1$$

## MOLE FRACTIONS




The number of moles of a nonreacting mixture is equal to the sum of the number of moles of its components

En la ppt Moles tambien es representada por: n

$$N_m = \sum_{i=1}^{k} N_i$$

$$y_i = \frac{N_i}{N_m}$$

$$y_{O_2} = \frac{1 \text{ kmol}}{4 \text{ kmol}} = 0.25$$

$$y_{H_2} = \frac{3 \text{ kmol}}{4 \text{ kmol}} = 0.75$$

and $$\sum_{i=1}^{k} y_i = 1$$

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# Mass and mole fractions of a mixture are related by

$$N = \frac{m}{M}$$

$$\text{mf}_i = \frac{m_i}{m_m} = \frac{N_i M_i}{N_m M_m} = y_i \frac{M_i}{M_m}$$

$y_i$

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## For ideal pure gas we use M to convert g ↔ mol

- And for mixture??

### We need to calculate $M_m$ (average molar mass )

$$M_m = \frac{m_m}{N_m} = \frac{\sum m_i}{\sum N_i}$$

$$M_m = \frac{\sum m_i}{N_m} = \frac{\sum N_i M_i}{N_m} = \sum y_i M_i$$

$$M_m = y_1 M_1 + y_2 M_2$$

$$= M(y_1 + y_2)$$

$$M_m = M \Rightarrow m_{fi} = y_i \frac{M_i}{M_m}$$

$$m_{fi} = y_i$$

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$$c_{v,m} = \sum_{i=1}^{k} \text{mf}_i c_{v,i} \quad (\text{kJ/kg} \cdot \text{K})$$

$$c_{p,m} = \sum_{i=1}^{k} \text{mf}_i c_{p,i} \quad (\text{kJ/kg} \cdot \text{K})$$

$$\bar{c}_{v,m} = \sum_{i=1}^{k} y_i \bar{c}_{v,i} \quad (\text{kJ/kmol} \cdot \text{K})$$

$$\bar{c}_{p,m} = \sum_{i=1}^{k} y_i \bar{c}_{p,i} \quad (\text{kJ/kmol} \cdot \text{K})$$

$c_p = c_v + R \longrightarrow c_{p,m} = c_{v,m} + R_m$, $\quad R_m = \dfrac{\bar{R}}{M_m}$

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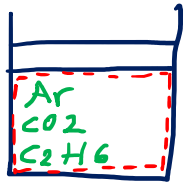
**EXAMPLE 1.3:** A gas mixture has the following composition on a mole basis: 40% Ar, 40 % $CO_2$ and 20 % $C_2H_6$. Determine mass fraction, average molar mass (**kg/kmol**), and gas constant of the mixture (**kJ/kg.K**).




$y_A \rightsquigarrow m_{FA}$ ?
$y_B \rightsquigarrow m_{FB}$ ?
$y_C \rightsquigarrow m_{FC}$ ?

$M_m$ ?

$R_m$ ?

$$\frac{\text{moles de A}}{\text{moles totales}} \times 100 = \%\ A$$

**TABLE A-1**

**Atomic or Molecular Weights and Critical Properties of Selected Elements and Compounds**

| Substance | Chemical Formula | $M$ (kg/kmol) | $T_c$ (K) | $p_c$ (bar) | $Z_c = \frac{p_c v_c}{R T_c}$ |
|---|---|---|---|---|---|
| Acetylene | $C_2H_2$ | 26.04 | 309 | 62.8 | 0.274 |
| Air (equivalent) | — | 28.97 | 133 | 37.7 | 0.284 |
| Ammonia | $NH_3$ | 17.03 | 406 | 112.8 | 0.242 |
| Argon | Ar | 39.94 | 151 | 48.6 | 0.290 |
| Benzene | $C_6H_6$ | 78.11 | 563 | 49.3 | 0.274 |
| Butane | $C_4H_{10}$ | 58.12 | 425 | 38.0 | 0.274 |
| Carbon | C | 12.01 | — | — | — |
| Carbon dioxide | $CO_2$ | 44.01 | 304 | 73.9 | 0.276 |
| Carbon monoxide | CO | 28.01 | 133 | 35.0 | 0.294 |
| Copper | Cu | 63.54 | — | — | — |
| Ethane | $C_2H_6$ | 30.07 | 305 | 48.8 | 0.285 |
| Ethanol | $C_2H_5OH$ | 46.07 | 516 | 63.8 | 0.249 |
| Ethylene | $C_2H_4$ | 28.05 | 283 | 51.2 | 0.270 |
| Helium | He | 4.003 | 5.2 | 2.3 | 0.300 |
| Hydrogen | $H_2$ | 2.016 | 33.2 | 13.0 | 0.304 |
| Methane | $CH_4$ | 16.04 | 191 | 46.4 | 0.290 |
| Methanol | $CH_3OH$ | 32.04 | 513 | 79.5 | 0.220 |
| Nitrogen | $N_2$ | 28.01 | 126 | 33.9 | 0.291 |

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- **Molar mass of the mixture**:

$$M_m = \sum_{i=1}^{n} y_i M_i = y_{Ar} M_{Ar} + y_{CO_2} M_{CO_2} + y_{C_2H_6} M_{C_2H_6}$$

$$M_m = 0.40 \times 39.94 + 0.40 \times 44.01 + 0.20 \times 30.07 = 39.594 \frac{kg}{kmol}$$

- **Mass fractions of the mixture**:

$m_{fi} = \frac{y_i M_i}{M_m}$,  $m_{f_{Ar}} = 0.4 * \frac{39.94}{39.594} = 0.4035$

$m_{f_{CO_2}} = 0.4 * \frac{44.01}{39.594} = 0.4446$

$m_{f_{C_2H_6}} = 0.2 * \frac{30.07}{39.594} = 0.1519$

- **Constant gas for of the mixture**:

$$R_m = \frac{\bar{R}}{M_m} = \frac{8.314\, kJ/kmol.K}{39.594 kg/kmol} = 0.210 \frac{kJ}{kg.K}$$

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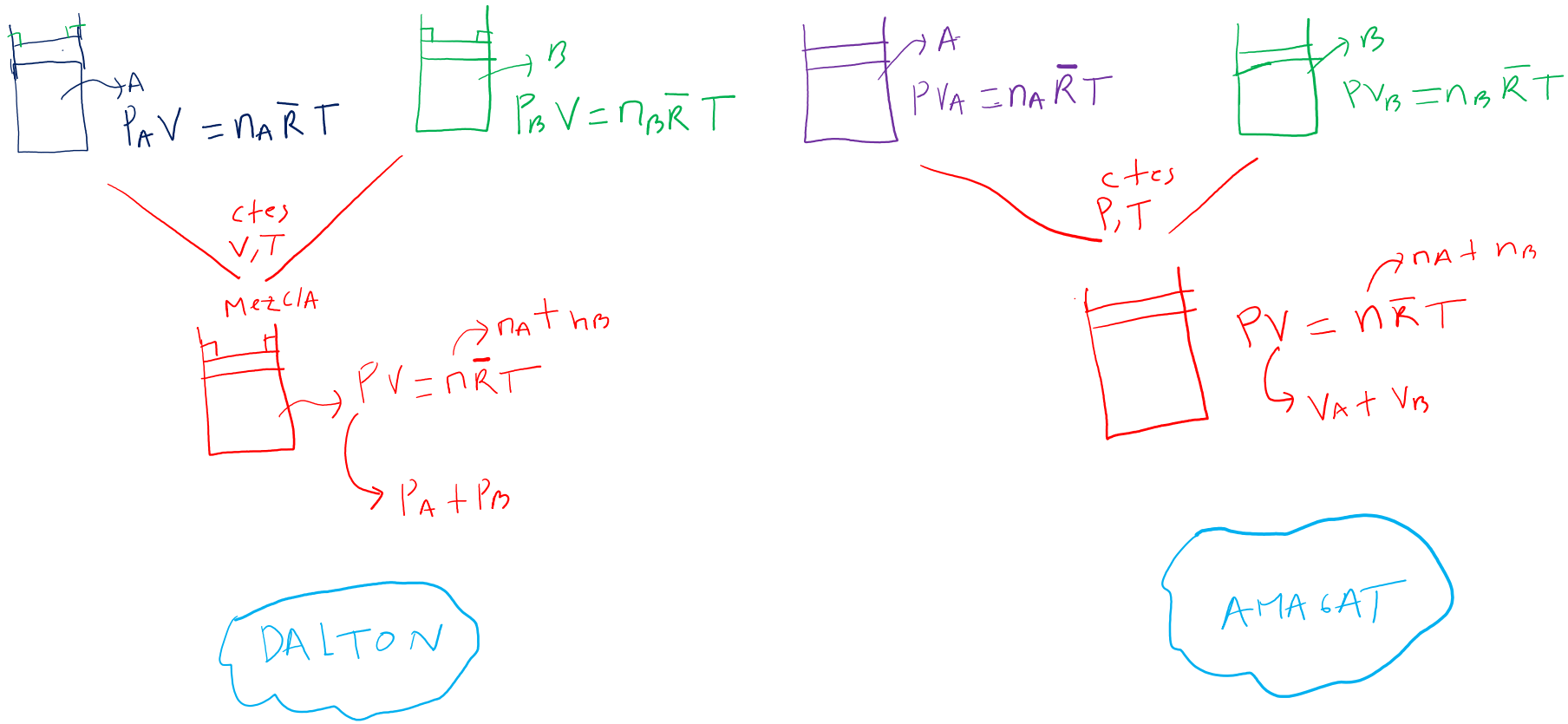
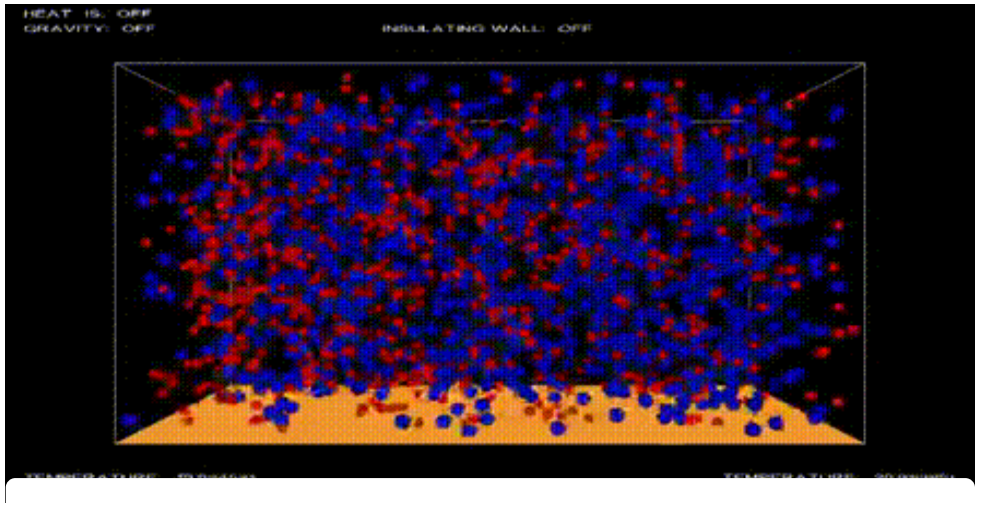
# *P-v-T* BEHAVIOR OF GAS MIXTURES: IDEAL GASES



# *P-v-T* BEHAVIOR OF GAS MIXTURES: IDEAL GASES




- ✓ When two or more ideal gases are mixed, the behavior of a molecule normally is not influenced by the presence of other similar or dissimilar molecules, and therefore a nonreacting mixture of ideal gases also behaves as an ideal gas.

- ✓ Air, for example, is conveniently treated as an ideal gas in the range where $N_2$ and $O_2$ behave as ideal gases.

https://www.youtube.com/watch?v=tEFHkcx2cz0

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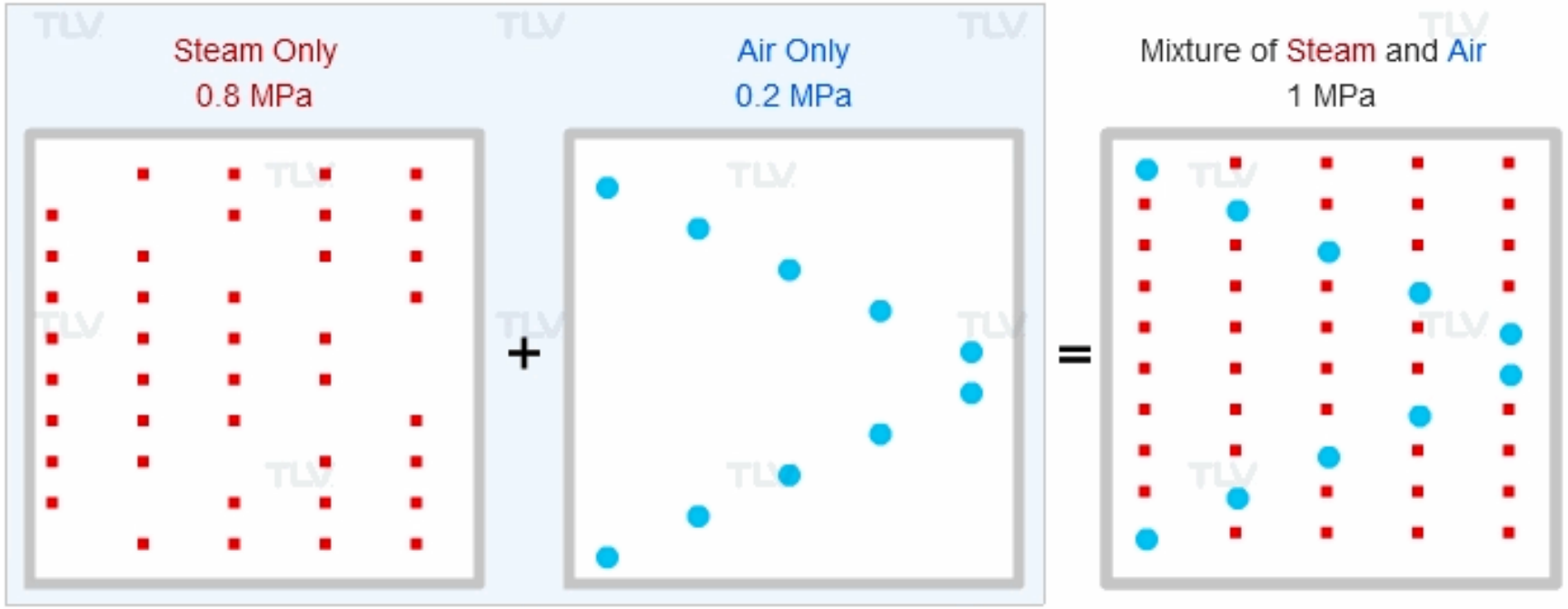
# *P-v-T* BEHAVIOR OF GAS MIXTURES: IDEAL GASES

***Dalton's law of additive pressures:*** *The total pressure of a gas mixture is equal to the sum of the partial pressures that each individual gas would exert if it were alone in the same volume and at the same temperature, assuming constant temperature and volume conditions.*

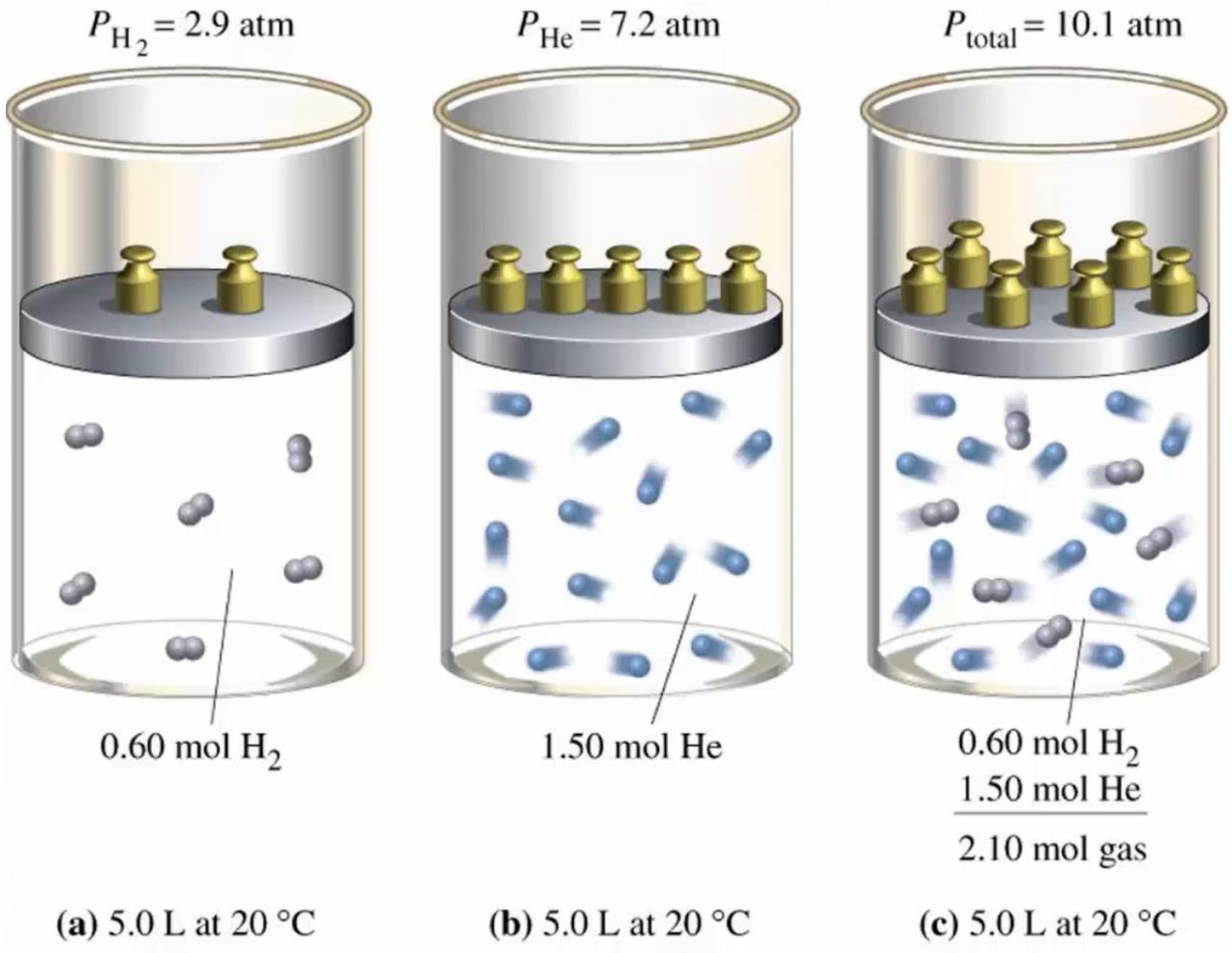



T, V = cte

$$P_m = P_1 + P_2$$

$$P_m = P_A + P_B$$

https://www.tlv.com/global/ME/steam-theory/temperature-problems-caused-by-air.html




https://www.youtube.com/watch?v=YUiMHXDjpWw

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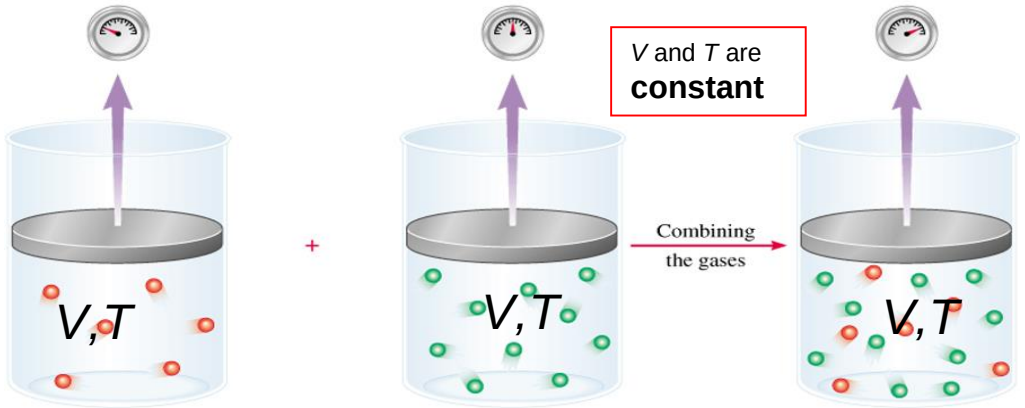
# Dalton's Law of Partial Pressures




$P_A$ $P_B$ $P_m = P_A + P_B$

$$\frac{\eta_A}{\eta_m} = \frac{P_A V_m / \bar{R} T_m}{P_m V_m / \bar{R} T_m} = \frac{P_A}{P_m} = y_A$$

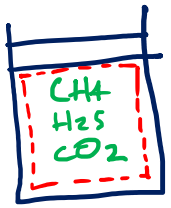
$$P_A = y_A P_m \qquad P_B = y_B P_m$$

$$P_i = y_i P_m$$

## EXAMPLE 2.4: Applying Dalton's Law of Partial Pressures

moles A / moles totales x 100 = % A

A refinery gas mixture has 50% $CH_4$, 10% $H_2S$ and 40%$CO_2$ on molar basis. Assume a total pressure of 250 kPa, find the partial pressures of each component [kPa]




- Mol fractions of each component

$$y_{CH_4} = 50\% = 0.50 \qquad y_{H_2S} = 10\% = 0.10 \qquad y_{CO_2} = 40\% = 0.40$$

- **Partial pressures**:

$$p_{CH_4} = y_{CH_4}P = 0.50 \times 250 = 125\ kPa$$

$$p_{H_2S} = y_{H_2S}P = 0.10 \times 250 = 25\ kPa$$

$$p_{CO_2} = y_{CO_2}P = 0.40 \times 250 = 100\ kPa$$

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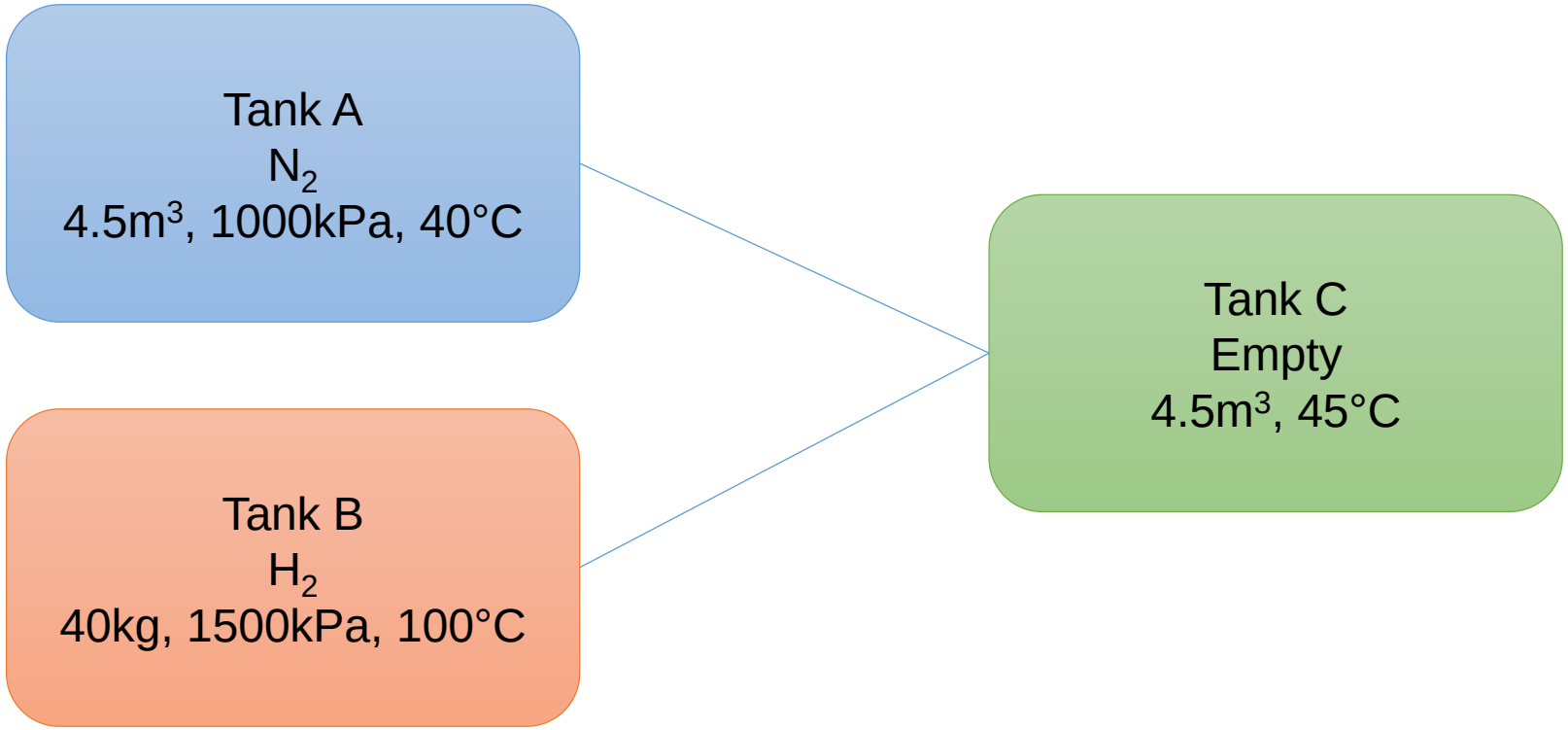
# EXAMPLE 3.5: Applying Partial Pressure_Ideal gases

Two tanks, A and B, are connected to a third tank C initially empty. Tank A has a volume of 4.5$m^3$ and contains nitrogen at 1000kPa and 40°C. Tank B contains 40kg of hydrogen at 1500kPa and 100°C. The valve that connects A and the empty tank C is opened until two thirds of the mass is transferred. Finally, the valve that connects B and C is opened until the pressure in C reaches 900kPa. If the tank C is rigid, has a volume of 4.5$m^3$ and remains at 45°C during the whole operation, estimate the molar fraction of $N_2$, the mass fraction of $H_2$, and the partial pressures (kPa) of each component inside tank C when the process ends.



**Tank A**

Tank A
$N_2$
4.5$m^3$, 1000kPa, 40°C

$$PV = mRT$$

$$1000 \times 4.5 = m \times \frac{8.314}{28.01} \times (40 + 273.15)$$

$$m = m_{N2\_A1} = 48.4131\ kg$$

**Tank B**

Tank B
$H_2$
40kg, 1500kPa, 100°C

$$PV = mRT$$

$$1500 \times V = 40 \times \frac{8.314}{2.016} \times (100 + 273.15)$$

$$V = 41.0366\ m3$$

# Tank C – final state

Tank C
Is not Empty
4.5m$^3$, 45°C
900kPa

$$PV = n\overline{R}T$$

$$900 \times 4.5 = n \times 8.314 \times (45 + 273.15)$$

$$n = 1.5311\ kmol$$

There is a mixture inside tank C, this mixture contains $\frac{2}{3} \times 48.4131 = 32.2754$ kg of nitrogen and an unknown amount of hydrogen:

$$n = \frac{m_{N_2}}{M_{N_2}} + \frac{m_{H_2}}{M_{H_2}}$$

$$1.5311 = \frac{32.2754}{28.01} + \frac{m_{H_2}}{2.016} \longrightarrow \boxed{m_{H_2} = 0.7637\ kg}$$

## Tank C – final state

**Molar fraction:**

$$\boldsymbol{y_{N2}} = \frac{n_{N2}}{n} = \frac{32.2754/28.01}{1.5311} = 0.7526$$

**Mass fraction:**

$$\boldsymbol{mf_{H2}} = \frac{m_{H2}}{m_{H2} + m_{N2}} = \frac{0.7637}{0.7637 + 32.2754} = 0.02312$$

**Partial pressure:**

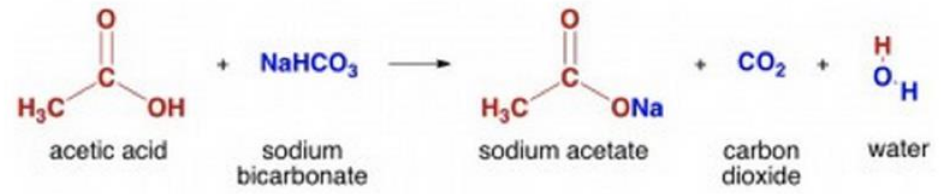
$$\boldsymbol{p_{N2}} = 0.7526 \times 900$$

$$\boldsymbol{p_{N2}} = 677.34\ kPa$$

$$\boldsymbol{p_{H2}} = (1 - 0.7526) \times 900$$

$$\boldsymbol{p_{H2}} = 222.66\ kPa$$

# Amagat’s Law of Partial Volumen




https://www.youtube.com/watch?v=KPSX5M-Isvk

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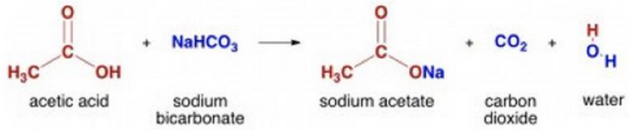
# Amagat’s Law of Partial Volumen

$$H_2O_2 + NaCl\,O \longrightarrow O_2 + NaCl + H_2O$$


https://www.youtube.com/watch?v=oo4hSiYxff8

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# Amagat's Law of Partial Volumen

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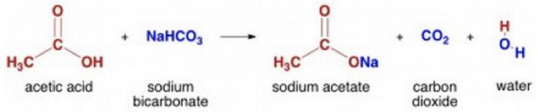


https://www.youtube.com/watch?v=oo4hSiYxff8

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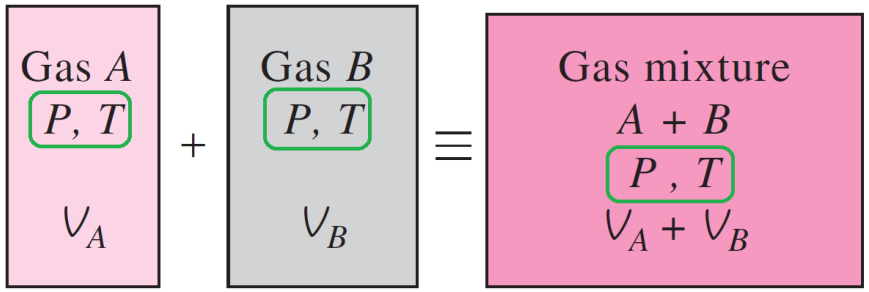
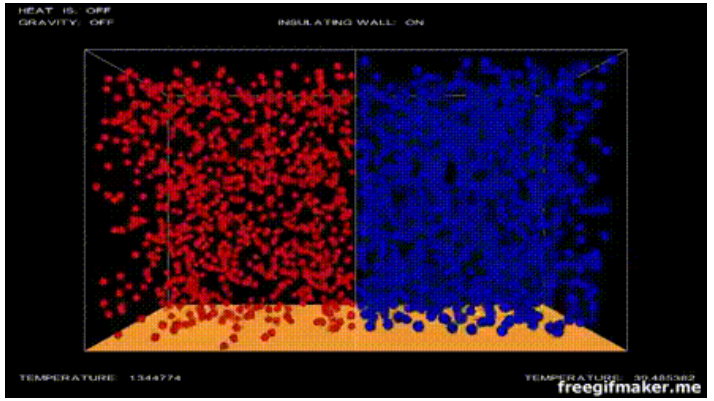
# Amagat's Law of Partial Volumen




$V_A = \eta_A RT_m/P_m$ and $V_m = V_A + V_B + \ldots$

$$\frac{\mathbf{V_A}}{\mathbf{V_m}} = \frac{\eta_A \bar{R} T_m/P_m}{\eta_m \bar{R} T_m/P_m} = \frac{\eta_A}{\eta_m} = \mathbf{y_A}$$

Same way to $\mathbf{y_B}$.....

$$V_i / V_m = y_i$$

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# Orsat Analysis

# EXAMPLE 4.2: Applying Amagat's Law of Partial Volumes

$\frac{V_A}{V_T} \times 100 = \%A$

A process requires a mixture that is 10 % acetylene, 60 % argon, and 30 % helium by volume. All three gases are supplied from separate tanks to an adiabatic, constant-pressure mixing chamber at 500 kPa. Determine the molar fraction, partial pressure (kpa), mass fraction of each component and the average molar mass of the mixture(kg/kmol).

**TABLE A-1**

**Atomic or Molecular Weights and Critical Properties of Selected Elements and Compounds**

| Substance | Chemical Formula | $M$ (kg/kmol) | $T_c$ (K) | $p_c$ (bar) | $Z_c = \frac{p_c v_c}{R T_c}$ |
|---|---|---|---|---|---|
| Acetylene | $C_2H_2$ | 26.04 | 309 | 62.8 | 0.274 |
| Air (equivalent) | — | 28.97 | 133 | 37.7 | 0.284 |
| Ammonia | $NH_3$ | 17.03 | 406 | 112.8 | 0.242 |
| Argon | Ar | 39.94 | 151 | 48.6 | 0.290 |
| Benzene | $C_6H_6$ | 78.11 | 563 | 49.3 | 0.274 |
| Butane | $C_4H_{10}$ | 58.12 | 425 | 38.0 | 0.274 |
| Carbon | C | 12.01 | — | — | — |
| Carbon dioxide | $CO_2$ | 44.01 | 304 | 73.9 | 0.276 |
| Carbon monoxide | CO | 28.01 | 133 | 35.0 | 0.294 |
| Copper | Cu | 63.54 | — | — | — |
| Ethane | $C_2H_6$ | 30.07 | 305 | 48.8 | 0.285 |
| Ethanol | $C_2H_5OH$ | 46.07 | 516 | 63.8 | 0.249 |
| Ethylene | $C_2H_4$ | 28.05 | 283 | 51.2 | 0.270 |
| Helium | He | 4.003 | 5.2 | 2.3 | 0.300 |
| Hydrogen | $H_2$ | 2.016 | 33.2 | 13.0 | 0.304 |
| Methane | $CH_4$ | 16.04 | 191 | 46.4 | 0.290 |
| Methanol | $CH_3OH$ | 32.04 | 513 | 79.5 | 0.220 |
| Nitrogen | $N_2$ | 28.01 | 126 | 33.9 | 0.291 |
| Octane | $C_8H_{18}$ | 114.22 | 569 | 24.9 | 0.258 |
| Oxygen | $O_2$ | 32.00 | 154 | 50.5 | 0.290 |
| Propane | $C_3H_8$ | 44.09 | 370 | 42.7 | 0.276 |

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**Note that volume fractions are equal to mole fractions in ideal gas mixtures( from Amagat´s law): ( $y_A = V_A/V$), therefore:**

$$y_{C_2H_2} = 10\% = 0.10 \qquad y_{Ar} = 60\% = 0.60 \qquad y_{He} = 30\% = 0.30$$

- **Partial pressures:**

$$p_{C_2H_2} = y_{C_2H_2}P = 0.10 \times 500 = 50kPa \qquad p_{Ar} = y_{Ar}P = 0.60 \times 500 = 300kPa$$

$$p_{He} = y_{He}P = 0.30 \times 500 = 150kPa$$

- **Molar mass of mixture**:

$$M_m = \sum_{i=1}^{n} y_i M_i = y_{C_2H_2}M_{C_2H_2} + y_{Ar}M_{Ar} + y_{He}M_{He}$$

$$M_m = 0.10 \times 26.04 + 0.60 \times 39.94 + 0.30 \times 4.003 = 27.7689 \frac{kg}{kmol}$$

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- **Mass fractions:**

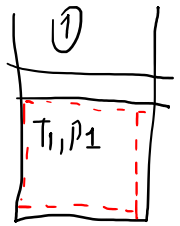
$$mf_i = y_i \frac{M_i}{M_m}$$

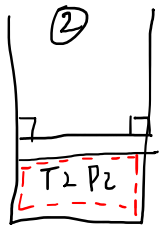
$$mf_{C_2H_2} = y_{C_2H_2} \frac{M_{C_2H_2}}{M_m} = 0.10 \times \frac{26.04}{27.7689} = 0.0938 \qquad mf_{Ar} = y_{Ar} \frac{M_{Ar}}{M_m} = 0.60 \times \frac{39.94}{27.7689} = 0.863$$

$$mf_{He} = y_{He} \frac{M_{He}}{M_m} = 0.30 \times \frac{4.003}{27.7689} = 0.0432$$

- Apply the rules for determining mixture properties (**u, h**) to ideal gas mixtures

# Masa de control – mezclas




$u_{1m}, h_{1m}$

$m_A + m_B = m_m$

$u_{2m}, h_{2m}$

$m_A + m_B = m_m$

$\Delta u_{12}$ ?

$\Delta u_{12} = u_{2m} - u_{1m}$

$u_{2m} = m_{FA} u_{A2} + m_{FB} u_{B2} = \sum m_{Fi} u_i$

$u_{1m} = m_{FA} u_{A1} + m_{FB} u_{B1} = \sum m_{Fi} u_i$

$\Delta u_{12} = m_{FA}(u_{A2} - u_{A1}) + m_{FB}(u_{B2} - u_{B1})$

$= \sum m_{Fi} \Delta u_i$

$\Delta U$ ? kJ

$\Delta U = m_m \Delta u_{12}$

$= m_A(u_{A2} - u_{A1}) + m_B(u_{B2} - u_{B1}) = \sum m_i \Delta u_i$

$\Delta \bar{u} = \sum y_i \Delta \bar{u}_i$

$\Delta U = N_m(\Delta \bar{u}) = \sum N_i \Delta \bar{u}_i$

Todo lo anterior se extrapola para la entalpía. $\begin{cases} * h \\ * \bar{h} \end{cases}$

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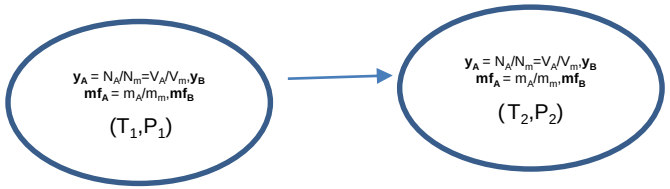
# OTHER STATE FUNCTIONS (PROPERTIES) OF IDEAL GAS MIXTURES_intensive

$$u_m = \sum_{i=1}^{k} \text{mf}_i u_i \quad (\text{kJ/kg}) \qquad \text{and} \qquad \bar{u}_m = \sum_{i=1}^{k} y_i \bar{u}_i \quad (\text{kJ/kmol})$$

$$h_m = \sum_{i=1}^{k} \text{mf}_i h_i \quad (\text{kJ/kg}) \qquad \text{and} \qquad \bar{h}_m = \sum_{i=1}^{k} y_i \bar{h}_i \quad (\text{kJ/kmol})$$

The changes in internal energy, enthalpy, and entropy can be expressed, respectively as:(CLOSED SYTEM)




$$\Delta u_m = \sum_{i=1}^{k} \text{mf}_i \Delta u_i \quad (\text{kJ/kg}) \qquad \text{and} \quad \Delta \bar{u}_m = \sum_{i=1}^{k} y_i \Delta \bar{u}_i \quad (\text{kJ/kmol})$$

$$\Delta h_m = \sum_{i=1}^{k} \text{mf}_i \Delta h_i \quad (\text{kJ/kg}) \qquad \text{and} \quad \Delta \bar{h}_m = \sum_{i=1}^{k} y_i \Delta \bar{h}_i \quad (\text{kJ/kmol})$$

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# OTHER STATE FUNCTIONS (PROPERTIES) OF IDEAL GAS MIXTURES_extensive

$$U_m = \sum_{i=1}^{k} U_i = \sum_{i=1}^{k} m_i u_i = \sum_{i=1}^{k} N_i \bar{u}_i \qquad (\text{kJ})$$

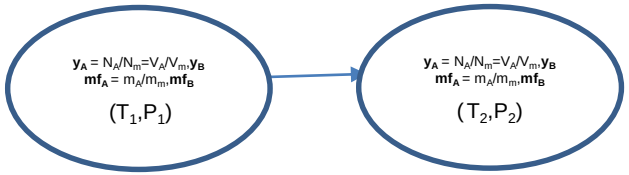
$$H_m = \sum_{i=1}^{k} H_i = \sum_{i=1}^{k} m_i h_i = \sum_{i=1}^{k} N_i \bar{h}_i \qquad (\text{kJ})$$

$U_m = m_m (m_{FA} u_A + m_{FB} u_B)$

$m_{FA} = \frac{m_A}{m_m}$

$U_m = m_A u_A + m_B u_B$

The changes in internal energy, enthalpy, and entropy can be expressed, respectively as: (CLOSED SYTEM)




$$\Delta U_m = \sum_{i=1}^{k} \Delta U_i = \sum_{i=1}^{k} m_i \, \Delta u_i = \sum_{i=1}^{k} N_i \, \Delta \bar{u}_i \qquad (\text{kJ})$$

$$\Delta H_m = \sum_{i=1}^{k} \Delta H_i = \sum_{i=1}^{k} m_i \, \Delta h_i = \sum_{i=1}^{k} N_i \, \Delta \bar{h}_i \qquad (\text{kJ})$$

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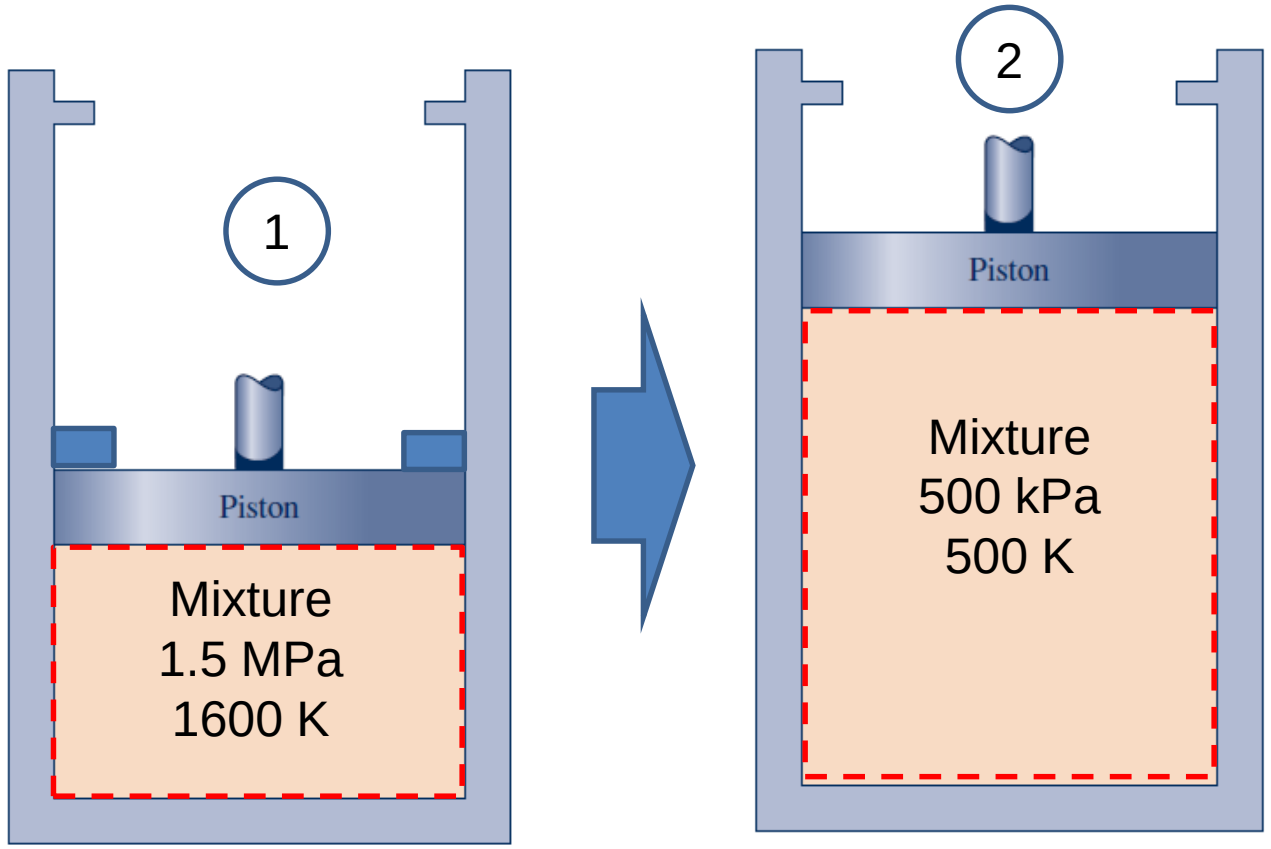
## EXAMPLE 5.2: $\Delta\bar{h}_m$ , $\Delta h_m$ and $\Delta u_m$ using tables.

A piston–cylinder device contains 250g of products of combustion. The combustion process results in a mixture that has the composition on a volume basis as follows: 20.00 percent carbon dioxide, 22.50 percent water vapor, 7.50 percent oxygen, and 50.00 percent nitrogen. This mixture is initially at 1600 K and 1.5 MPa and expands in an adiabatic, reversible process to 500 kPa and 500 K. Determine the molar fraction, the average molar mass , the mass fraction, the specific internal energy change: $\Delta u_m$ [**kJ/kg**] and the total enthalpy change: $\Delta H_m$ [**kJ**]. Use table A-23.




Molar mass from Table A-1:

- $M_{CO_2} = 44.01\ kg/kmol$
- $M_{H_2O} = 18.02\ kg/kmol$
- $M_{O_2} = 32.00\ kg/kmol$
- $M_{N_2} = 28.01\ kg/kmol$

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**Note that volume fractions are equal to mole fractions in ideal gas mixtures (from Amagat´s law): ( $y_A = V_A/V$), therefore:**

$$y_{CO_2} = 20\% = 0.200 \qquad y_{H_2O} = 22.5\% = 0.225$$

$$y_{O_2} = 7.5\% = 0.075 \qquad y_{N_2} = 50\% = 0.500$$

- **<u>Molar mass of mixture</u>**:

$$M_m = \sum_{i=1}^{n} y_i M_i = y_{CO_2} M_{CO_2} + y_{H_2O} M_{H_2O} + y_{O_2} M_{O_2} + y_{N_2} M_{N_2}$$

$$M_m = 0.200 \times 44.01 + 0.225 \times 18.02 + 0.075 \times 32.00 + 0.500 \times 28.01$$

$$M_m = 29.2615 \ \frac{kg}{kmol}$$

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- **Mass fractions:** $mf_i = y_i \frac{M_i}{M_m}$

$$mf_{CO_2} = y_{CO_2} \frac{M_{CO_2}}{M_m} = 0.200 \times \frac{44.01}{29.2615} = 0.301$$

$$mf_{H_2O} = y_{H_2O} \frac{M_{H_2O}}{M_m} = 0.225 \times \frac{18.02}{29.2615} = 0.139$$

$$mf_{O_2} = y_{O_2} \frac{M_{O_2}}{M_m} = 0.075 \times \frac{32.0}{29.2615} = 0.0820$$

$$mf_{N_2} = y_{N_2} \frac{M_{N_2}}{M_m} = 0.500 \times \frac{28.01}{29.2615} = 0.479$$

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- **Specific internal energy change (kJ/kg):**

$$\Delta\bar{u}_m = y_{CO_2}\left(\bar{u}_{CO_2,2} - \bar{u}_{CO_2,1}\right) + y_{H_2O}\left(\bar{u}_{H_2O,2} - \bar{u}_{H_2O,1}\right) + y_{O_2}\left(\bar{u}_{O_2,2} - \bar{u}_{O_2,1}\right) + y_{N_2}\left(\bar{u}_{N_2,2} - \bar{u}_{N_2,1}\right)$$

Shapiro's Table A-23 reports specific internal energies in molar basis at different temperatures, so we should change them to mass basis using the molar mass of each component:

**TABLE A-23**

**Ideal Gas Properties of Selected Gases**

Enthalpy $\bar{h}(T)$ and internal energy $\bar{u}(T)$, in kJ/kmol. Absolute entropy at 1 atm $\bar{s}^0(T)$, in kJ/kmol · K.

| | Carbon Dioxide, $CO_2$ ($\bar{h}_f^0 = -393{,}520$ kJ/kmol) | | | Carbon Monoxide, CO ($\bar{h}_f^0 = -110{,}530$ kJ/kmol) | | | Water Vapor, $H_2O$ ($\bar{h}_f^0 = -241{,}820$ kJ/kmol) | | | Oxygen, $O_2$ ($\bar{h}_f^0 = 0$ kJ/kmol) | | | Nitrogen, $N_2$ ($\bar{h}_f^0 = 0$ kJ/kmol) | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $T$(K) | $\bar{h}$ | $\bar{u}$ | $\bar{s}^0$ | $\bar{h}$ | $\bar{u}$ | $\bar{s}^0$ | $\bar{h}$ | $\bar{u}$ | $\bar{s}^0$ | $\bar{h}$ | $\bar{u}$ | $\bar{s}^0$ | $\bar{h}$ | $\bar{u}$ | $\bar{s}^0$ | $T$(K) |
| 500 | 17,678 | 13,521 | 234.814 | 14,600 | 10,443 | 212.719 | 16,828 | 12,671 | 206.413 | 14,770 | 10,614 | 220.589 | 14,581 | 10,423 | 206.630 | 500 |
| 1600 | 76,944 | 63,741 | 295.901 | 51,053 | 37,750 | 250.592 | 62,748 | 49,445 | 253.513 | 52,961 | 39,658 | 260.333 | 50,571 | 37,268 | 244.028 | 1600 |

$$\Delta\bar{u}_m = y_{CO_2}\left(\bar{u}_{CO_2,2} - \bar{u}_{CO_2,1}\right) + y_{H_2O}\left(\bar{u}_{H_2O,2} - \bar{u}_{H_2O,1}\right) + y_{O_2}\left(\bar{u}_{O_2,2} - \bar{u}_{O_2,1}\right) + y_{N_2}\left(\bar{u}_{N_2,2} - \bar{u}_{N_2,1}\right)$$

$$\Delta\bar{u}_m = 0.2(13521 - 63741) + 0.225(12671 - 49445) + 0.075(10614 - 39658) + 0.5(10423 - 37268)$$

$$\Delta\bar{u}_m = -33918.95\,\frac{kJ}{kmol}$$

$$\Delta\boldsymbol{u_m} = -33918.95\,\frac{kJ}{kmol} \times \frac{1}{29.2615\,\frac{kg}{kmol}} = -1159.1665\,kJ/kg$$

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- **Specific internal energy change (kJ/kg):**

$$\Delta u_m = mf_{CO_2}\left(u_{CO_2,2} - u_{CO_2,1}\right) + mf_{H_2O}\left(u_{H_2O,2} - u_{H_2O,1}\right) + mf_{O_2}\left(u_{O_2,2} - u_{O_2,1}\right) + mf_{N_2}\left(u_{N_2,2} - u_{N_2,1}\right)$$

Shapiro's Table A-23 reports specific internal energies in molar basis at different temperatures, so we should change them to mass basis using the molar mass of each component:

**TABLE A-23**

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Enthalpy $\bar{h}(T)$ and internal energy $\bar{u}(T)$, in kJ/kmol. Absolute entropy at 1 atm $\bar{s}^0(T)$, in kJ/kmol · K.

| | Carbon Dioxide, $CO_2$ ($\bar{h}_f^0 = -393{,}520$ kJ/kmol) | | | Carbon Monoxide, CO ($\bar{h}_f^0 = -110{,}530$ kJ/kmol) | | | Water Vapor, $H_2O$ ($\bar{h}_f^0 = -241{,}820$ kJ/kmol) | | | Oxygen, $O_2$ ($\bar{h}_f^0 = 0$ kJ/kmol) | | | Nitrogen, $N_2$ ($\bar{h}_f^0 = 0$ kJ/kmol) | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $T$(K) | $\bar{h}$ | $\bar{u}$ | $\bar{s}^0$ | $\bar{h}$ | $\bar{u}$ | $\bar{s}^0$ | $\bar{h}$ | $\bar{u}$ | $\bar{s}^0$ | $\bar{h}$ | $\bar{u}$ | $\bar{s}^0$ | $\bar{h}$ | $\bar{u}$ | $\bar{s}^0$ | $T$(K) |
| 500 | 17,678 | 13,521 | 234.814 | 14,600 | 10,443 | 212.719 | 16,828 | 12,671 | 206.413 | 14,770 | 10,614 | 220.589 | 14,581 | 10,423 | 206.630 | 500 |
| 1600 | 76,944 | 63,741 | 295.901 | 51,053 | 37,750 | 250.592 | 62,748 | 49,445 | 253.513 | 52,961 | 39,658 | 260.333 | 50,571 | 37,268 | 244.028 | 1600 |

$$\Delta u_m = mf_{CO_2}\left(\bar{\boldsymbol{u}}_{CO_2,2} - \bar{\boldsymbol{u}}_{CO_2,1}\right)/M_{CO_2} + mf_{H_2O}\left(\bar{\boldsymbol{u}}_{H_2O,2} - \bar{\boldsymbol{u}}_{H_2O,1}\right)/M_{H_2O} + mf_{O_2}\left(\bar{\boldsymbol{u}}_{O_2,2} - \bar{\boldsymbol{u}}_{O_2,1}\right)/M_{O_2} + mf_{N_2}\left(\bar{\boldsymbol{u}}_{N_2,2} - \bar{\boldsymbol{u}}_{N_2,1}\right)/M_{N_2}$$

$$\Delta u_m = 0.301\left(\frac{13521 - 63741}{44.01}\right) + 0.139\left(\frac{12671 - 49445}{18.02}\right) + 0.082\left(\frac{10614 - 39658}{32.00}\right) + 0.479\left(\frac{10423 - 37268}{28.01}\right)$$

$$\Delta u_m = -\mathbf{1160.6368}\,\frac{\boldsymbol{kJ}}{\boldsymbol{kg}}$$

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- **Total enthalpy change of mixture (kJ)**:

Similarly as internal energy, we can calculate the specific enthalpy change using:

**TABLE A-23**

**Ideal Gas Properties of Selected Gases**

Enthalpy $\bar{h}(T)$ and internal energy $\bar{u}(T)$, in kJ/kmol. Absolute entropy at 1 atm $\bar{s}^{\circ}(T)$, in kJ/kmol · K.

| | Carbon Dioxide, $CO_2$ ($\bar{h}_f^{\circ} = -393{,}520$ kJ/kmol) | | | Carbon Monoxide, CO ($\bar{h}_f^{\circ} = -110{,}530$ kJ/kmol) | | | Water Vapor, $H_2O$ ($\bar{h}_f^{\circ} = -241{,}820$ kJ/kmol) | | | Oxygen, $O_2$ ($\bar{h}_f^{\circ} = 0$ kJ/kmol) | | | Nitrogen, $N_2$ ($\bar{h}_f^{\circ} = 0$ kJ/kmol) | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $T$(K) | $\bar{h}$ | $\bar{u}$ | $\bar{s}^{\circ}$ | $\bar{h}$ | $\bar{u}$ | $\bar{s}^{\circ}$ | $\bar{h}$ | $\bar{u}$ | $\bar{s}^{\circ}$ | $\bar{h}$ | $\bar{u}$ | $\bar{s}^{\circ}$ | $\bar{h}$ | $\bar{u}$ | $\bar{s}^{\circ}$ | $T$(K) |
| 500 | 17,678 | 13,521 | 234.814 | 14,600 | 10,443 | 212.719 | 16,828 | 12,671 | 206.413 | 14,770 | 10,614 | 220.589 | 14,581 | 10,423 | 206.630 | 500 |
| 1600 | 76,944 | 63,741 | 295.901 | 51,053 | 37,750 | 250.592 | 62,748 | 49,445 | 253.513 | 52,961 | 39,658 | 260.333 | 50,571 | 37,268 | 244.028 | 1600 |

$$\Delta h_m = mf_{CO_2}(\bar{h}_{CO_2,2} - \bar{h}_{CO_2,1})/M_{CO_2} + mf_{H_2O}(\bar{h}_{H_2O,2} - \bar{h}_{H_2O,1})/M_{H_2O} + mf_{O_2}(\bar{h}_{O_2,2} - \bar{h}_{O_2,1})/M_{O_2} + mf_{N_2}(\bar{h}_{N_2,2} - \bar{h}_{N_2,1})/M_{N_2}$$

$$\Delta h_m = 0.301\left(\frac{17678 - 76944}{44.01}\right) + 0.139\left(\frac{16828 - 62748}{18.02}\right) + 0.082\left(\frac{14770 - 52961}{32.00}\right) + 0.479\left(\frac{14581 - 50571}{28.01}\right)$$

$$\Delta h_m = -1472.8828\ \frac{kJ}{kg}$$

Since we have 250 g of mixture, the total enthalpy change is:

$$\Delta \boldsymbol{H}_m = \boldsymbol{m\Delta h_m} = \boldsymbol{0.25kg \times -1472.8828\ \frac{kJ}{kg} = -368.2207\ kJ}$$

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