Thermodynamic

2026-1

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NAUDY LOPEZ

S_4_Ideal Gas Mixture_4.2

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Recomendaciones generales:

- Leer antes de clase y entender los conceptos o preguntar si es necesario.
- Practicar bastante:
  - *Ejercicios de clase
  - *Asesorías
  - *Ejercicios del libro resueltos y los de final de capitulo
- Los quizzes virtuales y gradescope úsenlos para aprender de forma honesta (se puede inferir).
- Tengan grupos de estudio y apóyense mutuamente.
- Crear sus propias notas estudio y formulario.
- Son 4 horas de clase en la semana (asistir y ser puntuales, perder alguna es atrasarse mucho)
- Cada hora de clase son 3 horas de estudio por fuera mínimo.
- En la ppt de la semana 1 están todos los links claves para el curso.

Thermodynamics _ Evaluation

COMPONENTE	%
PARCIAL	20
FINAL	30
C	50
APROBACIÓN: 11/20	

SEMANA 1	TAREA VIRTUAL	SE CALIFICA AUTOMATICO NO NECESITA PROCEDIMIENTO
SEMANA 2	TAREA VIRTUAL	
SEMANA 3	TAREA VIRTUAL	.
SEMANA 4	**TAREA INDIVIDUAL 1** + EVALUACION_ AULA1	TAREA INDIVIDUAL: Esta parte incluye subir un archivo con la solución EVALUACION_ AULA: SEMANAS 1-2
SEMANA 5	TAREA VIRTUAL+ EVALUACION_ AULA2	EVALUACION_ AULA: SEMANAS 1-3
SEMANA 6	TAREA VIRTUAL**+TAREA INDIVIDUAL 2**	
SEMANA 7	TAREA VIRTUAL+ EVALUACION_ AULA3	EVALUACION_ AULA: SEMANAS 1-6
SEMANA 8	**PARCIAL**	
SEMANA 9	EVALUACION_ AULA4+**TAREA INDIVIDUAL 3 +Tarea grupal 1**	EVALUACION_ AULA: SEMANAS 1-8
SEMANA 10	TAREA VIRTUAL	
SEMANA 11	TAREA VIRTUAL	
SEMANA 12	TAREA VIRTUAL+ EVALUACION_ AULA5	EVALUACION_ AULA: SEMANAS 1-11
SEMANA 13	TAREA VIRTUAL**+TAREA INDIVIDUAL 4 + Tarea grupal 2**	
SEMANA 14	TAREA VIRTUAL+EVALUACION_ AULA6	EVALUACION_ AULA : SEMANAS 1-13
SEMANA 15		
SEMANA 16	**FINAL**	

***LAS TAREAS VIRTUALES O LOS GRADESCOPE (TAREA INDIVIDAL) se abren lunes 2pm y se cierran sábado media noche. La tarea virtual se presenta hasta un máximo de 3 veces.**

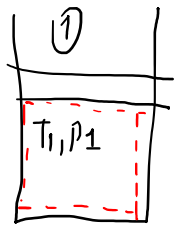
EVALUACION_ AULA: es un quiz de 20 min aprox. en aula.

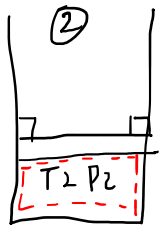
Session 7-8:

- Apply the ideal gas model for thermodynamic analysis of mixtures.

- Determine **u,h** for ideal gas mixtures.

Masa de control – mezclas





u_{1m}, h_{1m}

$m_A + m_B = m_m$

u_{2m}, h_{2m}

$m_A + m_B = m_m$

Δu_m ? [KJ/Kg]

$\Delta u_m = u_{2m} - u_{1m}$

$u_{2m} = m_{FA} u_{A2} + m_{FB} u_{B2}$

$u_{1m} = m_{FA} u_{A1} + m_{FB} u_{B1}$ $\Big\rangle \sum m_{Fi} u_i$

$\Delta u_m = m_{FA}(u_{A2} - u_{A1}) + m_{FB}(u_{B2} - u_{B1}) = \sum m_{Fi} \Delta u_i$

ΔU_m ? [KJ]

$\Delta U_m = m_m \Delta u_{12}$

$= m_A (u_{A2} - u_{A1}) + m_B (u_{B2} - u_{B1}) = \sum m_i \Delta u_i$

$\Delta \bar{u}_m = \sum y_i \Delta \bar{u}_i$ [KJ/KmoL]

$\Delta U_m = N_m (\Delta \bar{u}) = \sum N_i \Delta \bar{u}_i$ [KJ]

Todo lo anterior se extrapola para la entalpía. $\begin{cases} * h \\ * \bar{h} \end{cases}$

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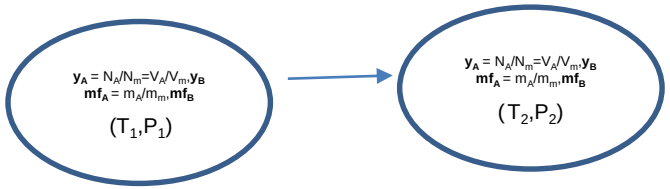
OTHER STATE FUNCTIONS (PROPERTIES) OF IDEAL GAS MIXTURES_intensive

$$u_m = \sum_{i=1}^{k} \text{mf}_i u_i \quad (\text{kJ/kg}) \qquad \text{and} \qquad \bar{u}_m = \sum_{i=1}^{k} y_i \bar{u}_i \quad (\text{kJ/kmol})$$

$$h_m = \sum_{i=1}^{k} \text{mf}_i h_i \quad (\text{kJ/kg}) \qquad \text{and} \qquad \bar{h}_m = \sum_{i=1}^{k} y_i \bar{h}_i \quad (\text{kJ/kmol})$$

The changes in internal energy, enthalpy, and entropy can be expressed, respectively as:(CLOSED SYTEM)



$$\Delta u_m = \sum_{i=1}^{k} \text{mf}_i \Delta u_i \quad (\text{kJ/kg}) \qquad \text{and} \quad \Delta \bar{u}_m = \sum_{i=1}^{k} y_i \Delta \bar{u}_i \quad (\text{kJ/kmol})$$

$$\Delta h_m = \sum_{i=1}^{k} \text{mf}_i \Delta h_i \quad (\text{kJ/kg}) \qquad \text{and} \quad \Delta \bar{h}_m = \sum_{i=1}^{k} y_i \Delta \bar{h}_i \quad (\text{kJ/kmol})$$

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OTHER STATE FUNCTIONS (PROPERTIES) OF IDEAL GAS MIXTURES_extensive

$$U_m = \sum_{i=1}^{k} U_i = \sum_{i=1}^{k} m_i u_i = \sum_{i=1}^{k} N_i \bar{u}_i \qquad (\text{kJ})$$

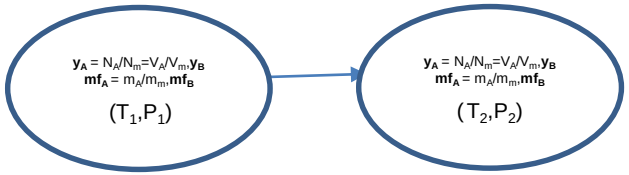
$$H_m = \sum_{i=1}^{k} H_i = \sum_{i=1}^{k} m_i h_i = \sum_{i=1}^{k} N_i \bar{h}_i \qquad (\text{kJ})$$

$U_m = m_m (m_{FA} u_A + m_{FB} u_B)$

$m_{FA} = \frac{m_A}{m_m}$

$U_m = m_A u_A + m_B u_B$

The changes in internal energy, enthalpy, and entropy can be expressed, respectively as: (CLOSED SYTEM)



$$\Delta U_m = \sum_{i=1}^{k} \Delta U_i = \sum_{i=1}^{k} m_i \, \Delta u_i = \sum_{i=1}^{k} N_i \, \Delta \bar{u}_i \qquad (\text{kJ})$$

$$\Delta H_m = \sum_{i=1}^{k} \Delta H_i = \sum_{i=1}^{k} m_i \, \Delta h_i = \sum_{i=1}^{k} N_i \, \Delta \bar{h}_i \qquad (\text{kJ})$$

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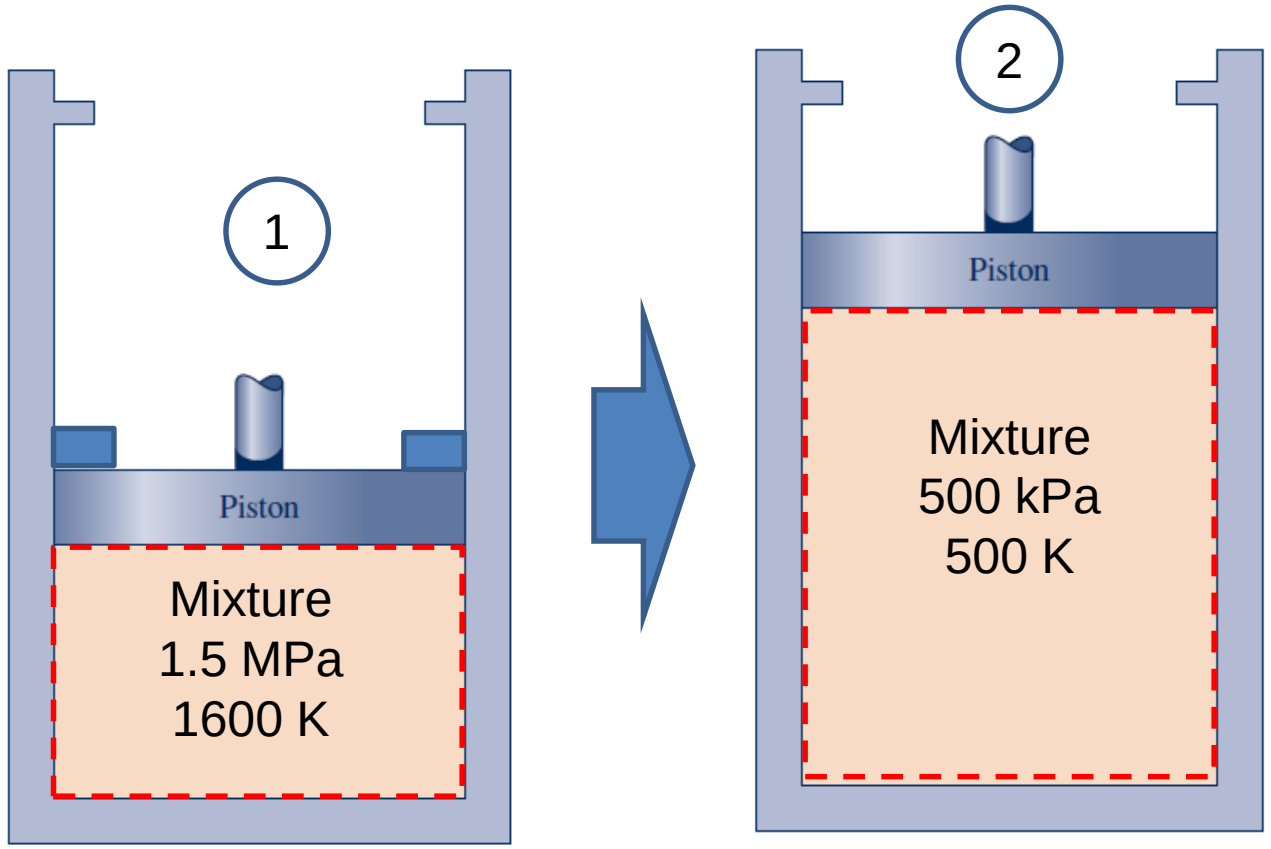
EXAMPLE 5.2: $\Delta\bar{h}_m$, Δh_m and Δu_m using tables.

A piston–cylinder device contains 250g of products of combustion. The combustion process results in a mixture that has the composition on a volume basis as follows: 20.00 percent carbon dioxide, 22.50 percent water vapor, 7.50 percent oxygen, and 50.00 percent nitrogen. This mixture is initially at 1600 K and 1.5 MPa and expands in an adiabatic, reversible process to 500 kPa and 500 K. Determine the molar fraction, the average molar mass , the mass fraction, the specific internal energy change: Δu_m [**kJ/kg**] and the total enthalpy change: ΔH_m [**kJ**]. Use table A-23.



Molar mass from Table A-1:

- $M_{CO_2} = 44.01\ kg/kmol$
- $M_{H_2O} = 18.02\ kg/kmol$
- $M_{O_2} = 32.00\ kg/kmol$
- $M_{N_2} = 28.01\ kg/kmol$

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Note that volume fractions are equal to mole fractions in ideal gas mixtures (from Amagat´s law): ($y_A = V_A/V$), therefore:

$$y_{CO_2} = 20\% = 0.200 \qquad y_{H_2O} = 22.5\% = 0.225$$

$$y_{O_2} = 7.5\% = 0.075 \qquad y_{N_2} = 50\% = 0.500$$

- **<u>Molar mass of mixture</u>**:

$$M_m = \sum_{i=1}^{n} y_i M_i = y_{CO_2} M_{CO_2} + y_{H_2O} M_{H_2O} + y_{O_2} M_{O_2} + y_{N_2} M_{N_2}$$

$$M_m = 0.200 \times 44.01 + 0.225 \times 18.02 + 0.075 \times 32.00 + 0.500 \times 28.01$$

$$M_m = 29.2615 \ \frac{kg}{kmol}$$

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- **Mass fractions:** $mf_i = y_i \frac{M_i}{M_m}$

$$mf_{CO_2} = y_{CO_2} \frac{M_{CO_2}}{M_m} = 0.200 \times \frac{44.01}{29.2615} = 0.301$$

$$mf_{H_2O} = y_{H_2O} \frac{M_{H_2O}}{M_m} = 0.225 \times \frac{18.02}{29.2615} = 0.139$$

$$mf_{O_2} = y_{O_2} \frac{M_{O_2}}{M_m} = 0.075 \times \frac{32.0}{29.2615} = 0.0820$$

$$mf_{N_2} = y_{N_2} \frac{M_{N_2}}{M_m} = 0.500 \times \frac{28.01}{29.2615} = 0.479$$

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- **Specific internal energy change (kJ/kg):**

$$\Delta\bar{u}_m = y_{CO_2}\left(\bar{u}_{CO_2,2} - \bar{u}_{CO_2,1}\right) + y_{H_2O}\left(\bar{u}_{H_2O,2} - \bar{u}_{H_2O,1}\right) + y_{O_2}\left(\bar{u}_{O_2,2} - \bar{u}_{O_2,1}\right) + y_{N_2}\left(\bar{u}_{N_2,2} - \bar{u}_{N_2,1}\right)$$

Shapiro's Table A-23 reports specific internal energies in molar basis at different temperatures, so we should change them to mass basis using the molar mass of each component:

TABLE A-23

Ideal Gas Properties of Selected Gases

Enthalpy $\bar{h}(T)$ and internal energy $\bar{u}(T)$, in kJ/kmol. Absolute entropy at 1 atm $\bar{s}^0(T)$, in kJ/kmol · K.

	Carbon Dioxide, CO_2 ($\bar{h}_f^0 = -393{,}520$ kJ/kmol)			Carbon Monoxide, CO ($\bar{h}_f^0 = -110{,}530$ kJ/kmol)			Water Vapor, H_2O ($\bar{h}_f^0 = -241{,}820$ kJ/kmol)			Oxygen, O_2 ($\bar{h}_f^0 = 0$ kJ/kmol)			Nitrogen, N_2 ($\bar{h}_f^0 = 0$ kJ/kmol)			
T(K)	$\bar{h}$	$\bar{u}$	$\bar{s}^0$	$\bar{h}$	$\bar{u}$	$\bar{s}^0$	$\bar{h}$	$\bar{u}$	$\bar{s}^0$	$\bar{h}$	$\bar{u}$	$\bar{s}^0$	$\bar{h}$	$\bar{u}$	$\bar{s}^0$	T(K)
500	17,678	13,521	234.814	14,600	10,443	212.719	16,828	12,671	206.413	14,770	10,614	220.589	14,581	10,423	206.630	500
1600	76,944	63,741	295.901	51,053	37,750	250.592	62,748	49,445	253.513	52,961	39,658	260.333	50,571	37,268	244.028	1600

$$\Delta\bar{u}_m = y_{CO_2}\left(\bar{u}_{CO_2,2} - \bar{u}_{CO_2,1}\right) + y_{H_2O}\left(\bar{u}_{H_2O,2} - \bar{u}_{H_2O,1}\right) + y_{O_2}\left(\bar{u}_{O_2,2} - \bar{u}_{O_2,1}\right) + y_{N_2}\left(\bar{u}_{N_2,2} - \bar{u}_{N_2,1}\right)$$

$$\Delta\bar{u}_m = 0.2(13521 - 63741) + 0.225(12671 - 49445) + 0.075(10614 - 39658) + 0.5(10423 - 37268)$$

$$\Delta\bar{u}_m = -33918.95\,\frac{kJ}{kmol}$$

$$\Delta\boldsymbol{u_m} = -33918.95\,\frac{kJ}{kmol} \times \frac{1}{29.2615\,\frac{kg}{kmol}} = -1159.1665\,kJ/kg$$

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- **Specific internal energy change (kJ/kg):**

$$\Delta u_m = mf_{CO_2}\left(u_{CO_2,2} - u_{CO_2,1}\right) + mf_{H_2O}\left(u_{H_2O,2} - u_{H_2O,1}\right) + mf_{O_2}\left(u_{O_2,2} - u_{O_2,1}\right) + mf_{N_2}\left(u_{N_2,2} - u_{N_2,1}\right)$$

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	Carbon Dioxide, CO_2 ($\bar{h}_f^0 = -393{,}520$ kJ/kmol)			Carbon Monoxide, CO ($\bar{h}_f^0 = -110{,}530$ kJ/kmol)			Water Vapor, H_2O ($\bar{h}_f^0 = -241{,}820$ kJ/kmol)			Oxygen, O_2 ($\bar{h}_f^0 = 0$ kJ/kmol)			Nitrogen, N_2 ($\bar{h}_f^0 = 0$ kJ/kmol)			
T(K)	$\bar{h}$	$\bar{u}$	$\bar{s}^0$	$\bar{h}$	$\bar{u}$	$\bar{s}^0$	$\bar{h}$	$\bar{u}$	$\bar{s}^0$	$\bar{h}$	$\bar{u}$	$\bar{s}^0$	$\bar{h}$	$\bar{u}$	$\bar{s}^0$	T(K)
500	17,678	13,521	234.814	14,600	10,443	212.719	16,828	12,671	206.413	14,770	10,614	220.589	14,581	10,423	206.630	500
1600	76,944	63,741	295.901	51,053	37,750	250.592	62,748	49,445	253.513	52,961	39,658	260.333	50,571	37,268	244.028	1600

$$\Delta u_m = mf_{CO_2}\left(\bar{\boldsymbol{u}}_{CO_2,2} - \bar{\boldsymbol{u}}_{CO_2,1}\right)/M_{CO_2} + mf_{H_2O}\left(\bar{\boldsymbol{u}}_{H_2O,2} - \bar{\boldsymbol{u}}_{H_2O,1}\right)/M_{H_2O} + mf_{O_2}\left(\bar{\boldsymbol{u}}_{O_2,2} - \bar{\boldsymbol{u}}_{O_2,1}\right)/M_{O_2} + mf_{N_2}\left(\bar{\boldsymbol{u}}_{N_2,2} - \bar{\boldsymbol{u}}_{N_2,1}\right)/M_{N_2}$$

$$\Delta u_m = 0.301\left(\frac{13521 - 63741}{44.01}\right) + 0.139\left(\frac{12671 - 49445}{18.02}\right) + 0.082\left(\frac{10614 - 39658}{32.00}\right) + 0.479\left(\frac{10423 - 37268}{28.01}\right)$$

$$\Delta u_m = -\mathbf{1160.6368}\,\frac{\boldsymbol{kJ}}{\boldsymbol{kg}}$$

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- **<u>Total enthalpy change of mixture (kJ)</u>**:

Similarly as internal energy, we can calculate the specific enthalpy change using:

TABLE A-23

Ideal Gas Properties of Selected Gases

Enthalpy $\bar{h}(T)$ and internal energy $\bar{u}(T)$, in kJ/kmol. Absolute entropy at 1 atm $\bar{s}^{\circ}(T)$, in kJ/kmol · K.

	Carbon Dioxide, CO_2 ($\bar{h}_f^{\circ}$ = −393,520 kJ/kmol)			Carbon Monoxide, CO ($\bar{h}_f^{\circ}$ = −110,530 kJ/kmol)			Water Vapor, H_2O ($\bar{h}_f^{\circ}$ = −241,820 kJ/kmol)			Oxygen, O_2 ($\bar{h}_f^{\circ}$ = 0 kJ/kmol)			Nitrogen, N_2 ($\bar{h}_f^{\circ}$ = 0 kJ/kmol)			
T(K)	$\bar{h}$	$\bar{u}$	$\bar{s}^{\circ}$	$\bar{h}$	$\bar{u}$	$\bar{s}^{\circ}$	$\bar{h}$	$\bar{u}$	$\bar{s}^{\circ}$	$\bar{h}$	$\bar{u}$	$\bar{s}^{\circ}$	$\bar{h}$	$\bar{u}$	$\bar{s}^{\circ}$	T(K)
500	17,678	13,521	234.814	14,600	10,443	212.719	16,828	12,671	206.413	14,770	10,614	220.589	14,581	10,423	206.630	500
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$$\Delta h_m = mf_{CO_2}(\bar{h}_{CO_2,2} - \bar{h}_{CO_2,1})/M_{CO_2} + mf_{H_2O}(\bar{h}_{H_2O,2} - \bar{h}_{H_2O,1})/M_{H_2O} + mf_{O_2}(\bar{h}_{O_2,2} - \bar{h}_{O_2,1})/M_{O_2} + mf_{N_2}(\bar{h}_{N_2,2} - \bar{h}_{N_2,1})/M_{N_2}$$

$$\Delta h_m = 0.301\left(\frac{17678 - 76944}{44.01}\right) + 0.139\left(\frac{16828 - 62748}{18.02}\right) + 0.082\left(\frac{14770 - 52961}{32.00}\right) + 0.479\left(\frac{14581 - 50571}{28.01}\right)$$

$$\Delta h_m = -1472.8828\ \frac{kJ}{kg}$$

Since we have <u>250 g</u> of mixture, the total enthalpy change is:

$$\Delta \boldsymbol{H}_m = \boldsymbol{m\Delta h}_m = \boldsymbol{0.25kg} \times \boldsymbol{-1472.8828}\ \frac{\boldsymbol{kJ}}{\boldsymbol{kg}} = \boldsymbol{-368.2207\ kJ}$$

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Masa de control — mezclas

$$\Delta U_m = m_{FA}(u_{A2} - u_{A1}) + m_{FB}(u_{B2} - u_{B1})$$

$$\downarrow$$

$$= \sum m_{Fi}\,\Delta u_i \ [KJ/Kg] = \sum m_{Fi}\,c_{vi}\,\Delta T = c_{vm}\,\Delta T$$

$$\Delta U_m = m_m\,\Delta u_{12} \ [KJ]$$

$$= m_A(u_{A2} - u_{A1}) + m_B(u_{B2} - u_{B1}) = \sum m_i\,\Delta u_i = \sum m_i\,c_{vi}\,\Delta T$$

$$= m_m\,c_{vm}\,\Delta T$$

$$\Delta \bar{u}_m = \sum y_i\,\Delta \bar{u}_i \ [KJ/KmoL]$$

$\sum y_i \bar{c}_{vi}\,\Delta T = \bar{c}_{vm}\,\Delta T$

$$\Delta U_m = N_m(\Delta \bar{u}) = \sum N_i\,\Delta \bar{u}_i \ [KJ]$$

$\sum N_i \bar{c}_{vi}\,\Delta T$

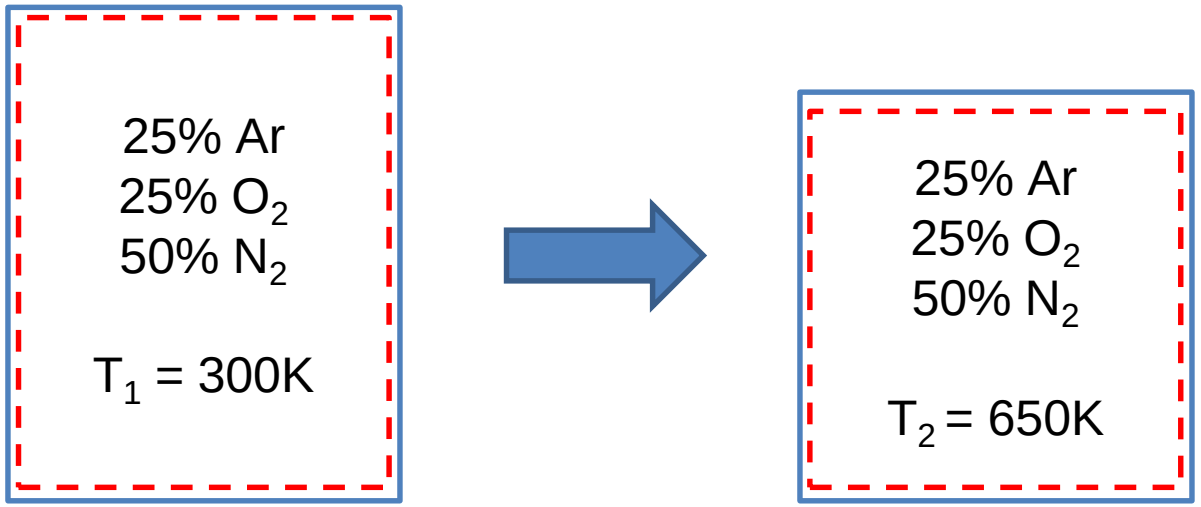
$N_m \bar{c}_{vm}\,\Delta T$

Todo lo anterior se extrapola para la entalpía. $\begin{cases} * h \\ * \bar{h} \end{cases}$

EXAMPLE 6.2: $\Delta\bar{h}_m$ using $\bar{c}_{p,m}$.

A mixture that is 25% argon, 25% oxygen, and 50% nitrogen by volume undergoes a process from 300 K and 200 kPa to 650 K and 500 kPa. Determine the average molar mass (kg/kmol), the mass fractions, and the enthalpy change per mol of mixture (kJ/kmol). Use the following constant specific heats for solving:

$$\bar{c}_{p,Ar} = 20.788\ kJ/kmolK, \quad \bar{c}_{p,O_2} = 30.150 kJ/kmolK, \quad \bar{c}_{p,N_2} = 30.558 kJ/kmolK$$



Note that volume fractions are equal to mole fractions in ideal gas mixtures (from Amagat´s law): ($y_A = V_A/V$), therefore:

$$y_{Ar} = 25\% = 0.25$$

$$y_{O_2} = 25\% = 0.25$$

$$y_{N_2} = 50\% = 0.50$$

- **<u>Molar mass of mixture</u>**:

$$M_m = \sum_{i=1}^{n} y_i M_i = y_{Ar} M_{Ar} + y_{O_2} M_{O_2} + y_{N_2} M_{N_2}$$

$$M_m = 0.25 \times 39.94 + 0.25 \times 32.00 + 0.50 \times 28.01$$

$$M_m = 31.99 \frac{kg}{kmol}$$

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- **Mass fractions:**

$$mf_i = y_i \frac{M_i}{M_m}$$

$$mf_{Ar} = y_{Ar} \frac{M_{Ar}}{M_m} = 0.25 \times \frac{39.94}{31.99} = 0.312$$

$$mf_{O_2} = y_{O_2} \frac{M_{O_2}}{M_m} = 0.25 \times \frac{32.0}{31.99} = 0.250$$

$$mf_{N_2} = y_{N_2} \frac{M_{N_2}}{M_m} = 0.50 \times \frac{28.01}{31.99} = 0.438$$

- **Specific enthalpy change (kJ/kmol):**

$$\Delta\bar{h}_m = y_{Ar}\left(\bar{h}_{Ar,2} - \bar{h}_{Ar,1}\right) + y_{O_2}\left(\bar{h}_{O_2,2} - \bar{h}_{O_2,1}\right) + y_{N_2}\left(\bar{h}_{N_2,2} - \bar{h}_{N_2,1}\right)$$

$$\Delta\bar{h}_m = y_{Ar}\bar{c}_{p,Ar}\,\Delta T + y_{O_2}\bar{c}_{p,O_2}\Delta T + y_{N_2}\bar{c}_{p,N_2}\Delta T$$

$$\Delta\bar{h}_m = \Delta T\left[y_{Ar}\bar{c}_{p,Ar} + y_{O_2}\bar{c}_{p,O_2} + y_{N_2}\bar{c}_{p,N_2}\right]$$

$$\Delta\bar{h}_m = \Delta T\left[\bar{c}_{p,m}\right]$$

$\bar{c}_{p,m}$ can be used when heat capacities are constant

$$\Delta\bar{h}_m = (650 - 300)K \times [0.25 \times 20.788 + 0.25 \times 30.150 + 0.50 \times 30.558]\ kJ/kmolK$$

$$\Delta\bar{h}_m = \mathbf{9804.725\ kJ/kmol}$$

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EXAMPLE 7.1….$\Delta\bar{u}_m$ (using tables), Δh_m (using $\bar{c}_p$(T)).

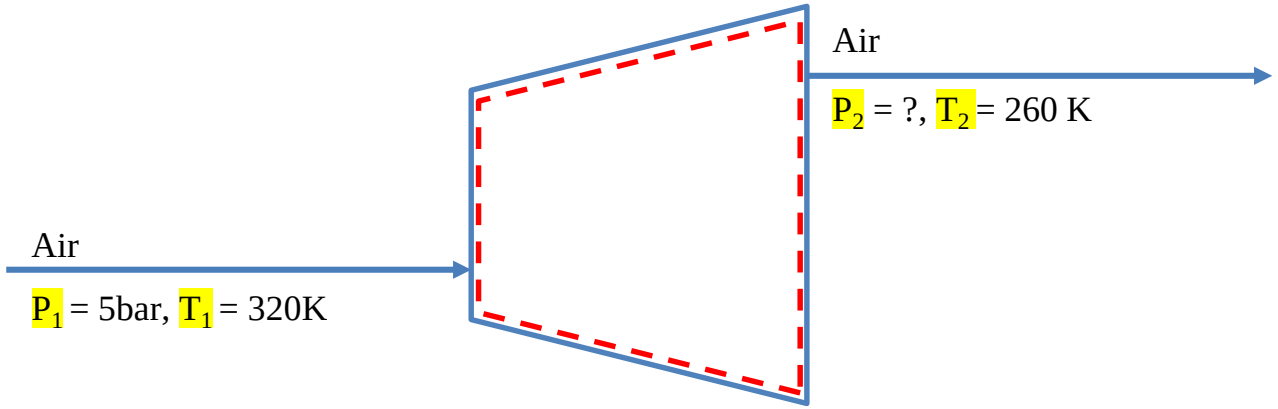
Modified air (50%mol N_2 and 50%mol O_2) is expanded in a turbine following a polytropic process ($P_1 v_1^k$ = CTE) with $k = 1.8$. The air enters the turbine at 5bar and 320 K and exits at 260 K. Calculate:

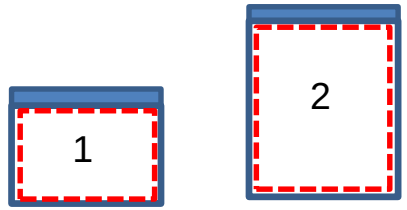
a) Mass fractions of air, average molar mass and pressure after the expansion
b) The change of specific internal energy (kJ/kmol) considering the properties shown in Table A-23
c) The change of specific enthalpy (kJ/kg) considering the following specific heat capacities (temperatures are in Kelvin):

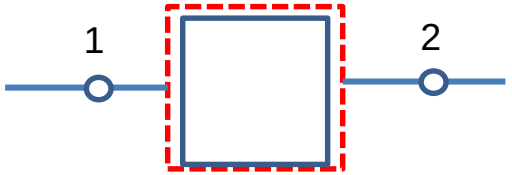
$\bar{c_{p,O_2/\bar{R}}} = (3.626 - 1.878 \times 10^{-3} T)$ … adimensional …$\bar{R}$ is the universal constant.. kJ/kmol.K

$\bar{c_{p,N_2/\bar{R}}} = (3.675 - 1.208 \times 10^{-3} T)$

NOTE: There is no chemical reaction in the turbine.







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a) Mass fractions, average molar mass and pressure

$$\boldsymbol{M_m} = \sum_{i=1}^{n} y_i M_i = y_{O_2} M_{O_2} + y_{N_2} M_{N_2} = 0.5 \times 32 + 0.5 \times 28.01 = 30.005\ kg/kmol$$

$$\boldsymbol{mf_{O_2}} = y_{O_2} \frac{M_{O_2}}{M_m} = 0.5 \times \frac{32}{30.01} = 0.533 \qquad \boldsymbol{mf_{N_2}} = y_{N_2} \frac{M_{N_2}}{M_m} = 0.5 \times \frac{28.01}{30.01} = 0.467$$

$$P_1 v_1^k = P_2 v_2^k$$

$$P_1 \left(\frac{\bar{R} T_1}{P_1 M}\right)^k = P_2 \left(\frac{\bar{R} T_2}{P_2 M}\right)^k$$

$$\frac{P_1 T_1^k}{P_1^k} = P_2 \left(\frac{T_2}{P_2}\right)^k \Leftrightarrow P_1^{(1-k)} T_1^k = P_2^{1-k} T_2^k$$

$$\left(\frac{P_1}{P_2}\right)^{1-k} = \left(\frac{T_2}{T_1}\right)^k$$

$$\left(\frac{5}{P_2}\right)^{1-1.8} = \left(\frac{260}{320}\right)^{1.8}$$

T_2 T_1

$$\boldsymbol{P_2} = 3.1338\ bar$$

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b) Specific internal energy change

TABLE A-23

Ideal Gas Properties of Selected Gases

Enthalpy $\bar{h}(T)$ and internal energy $\bar{u}(T)$, in kJ/kmol. Absolute entropy at 1 atm $\bar{s}^\circ(T)$, in kJ/kmol · K.

	Carbon Dioxide, CO_2 ($\bar{h}_f^\circ$ = −393,520 kJ/kmol)			Carbon Monoxide, CO ($\bar{h}_f^\circ$ = −110,530 kJ/kmol)			Water Vapor, H_2O ($\bar{h}_f^\circ$ = −241,820 kJ/kmol)			Oxygen, O_2 ($\bar{h}_f^\circ$ = 0 kJ/kmol)			Nitrogen, N_2 ($\bar{h}_f^\circ$ = 0 kJ/kmol)			
T(K)	$\bar{h}$	$\bar{u}$	$\bar{s}^\circ$	$\bar{h}$	$\bar{u}$	$\bar{s}^\circ$	$\bar{h}$	$\bar{u}$	$\bar{s}^\circ$	$\bar{h}$	$\bar{u}$	$\bar{s}^\circ$	$\bar{h}$	$\bar{u}$	$\bar{s}^\circ$	T(K)
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
220	6,601	4,772	202.966	6,391	4,562	188.683	7,295	5,466	178.576	6,404	4,575	196.171	6,391	4,562	182.638	220
230	6,938	5,026	204.464	6,683	4,771	189.980	7,628	5,715	180.054	6,694	4,782	197.461	6,683	4,770	183.938	230
240	7,280	5,285	205.920	6,975	4,979	191.221	7,961	5,965	181.471	6,984	4,989	198.696	6,975	4,979	185.180	240
250	7,627	5,548	207.337	7,266	5,188	192.411	8,294	6,215	182.831	7,275	5,197	199.885	7,266	5,188	186.370	250
260	7,979	5,817	208.717	7,558	5,396	193.554	8,627	6,466	184.139	7,566	5,405	201.027	7,558	5,396	187.514	260
270	8,335	6,091	210.062	7,849	5,604	194.654	8,961	6,716	185.399	7,858	5,613	202.128	7,849	5,604	188.614	270
280	8,697	6,369	211.376	8,140	5,812	195.173	9,296	6,968	186.616	8,150	5,822	203.191	8,141	5,813	189.673	280
290	9,063	6,651	212.660	8,432	6,020	196.735	9,631	7,219	187.791	8,443	6,032	204.218	8,432	6,021	190.695	290
298	9,364	6,885	213.685	8,669	6,190	197.543	9,904	7,425	188.720	8,682	6,203	205.033	8,669	6,190	191.502	298
300	9,431	6,939	213.915	8,723	6,229	197.723	9,966	7,472	188.928	8,736	6,242	205.213	8,723	6,229	191.682	300
310	9,807	7,230	215.146	9,014	6,437	198.678	10,302	7,725	190.030	9,030	6,453	206.177	9,014	6,437	192.638	310
320	10,186	7,526	216.351	9,306	6,645	199.603	10,639	7,978	191.098	9,325	6,664	207.112	9,306	6,645	193.562	320
330	10,570	7,826	217.534	9,597	6,854	200.500	10,976	8,232	192.136	9,620	6,877	208.020	9,597	6,853	194.459	330
340	10,959	8,131	218.694	9,889	7,062	201.371	11,314	8,487	193.144	9,916	7,090	208.904	9,888	7,061	195.328	340

T_2

T_1

$$\Delta\bar{u}_m = y_{O_2}\left(\bar{u}_{O_2,2} - \bar{u}_{O_2,1}\right) + y_{N_2}\left(\bar{u}_{N_2,2} - \bar{u}_{N_2,1}\right)$$

$$\Delta\bar{u}_m = 0.5(5405 - 6664) + 0.5(5396 - 6645)$$

$$\Delta\bar{u}_m = -1254\ kJ/kmol$$

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c) Specific enthalpy change

$$\overline{h}_{O_2,2} - \overline{h}_{O_2,1} = \int_{T_1}^{T_2} \overline{c}_{p,O_2} dT = \int_{320}^{260} (3.626 - 1.878 \times 10^{-3} T) dT \times \bar{R}$$

$$h_{O_2,2} - h_{O_2,1} = \left[3.626(260 - 320) - 1.878 \times 10^{-3} \left(\frac{260^2}{2} - \frac{320^2}{2}\right)\right] \times \frac{8.314}{32} = -48.035 \frac{kJ}{kg}$$

$$\overline{h}_{N_2,2} - \overline{h}_{N_2,1} = \int_{T_1}^{T_2} \overline{c}_{p,N_2} dT = \int_{320}^{260} (3.675 - 1.208 \times 10^{-3} T) dT \times \bar{R}$$

$$h_{N_2,2} - h_{N_2,1} = \left[3.675(260 - 320) - 1.208 \times 10^{-3} \left(\frac{260^2}{2} - \frac{320^2}{2}\right)\right] \times \frac{8.314}{28.01} = -59.210 \frac{kJ}{kg}$$

$$\Delta h_m = fm_{O_2}\left(h_{O_2,2} - h_{O_2,1}\right) + fm_{N_2}\left(h_{N_2,2} - h_{N_2,1}\right)$$

$$\Delta h_m = 0.533 \times -48.035 + 0.467 \times -59.210$$

$$\Delta h_m = -53.254\ kJ/kg$$

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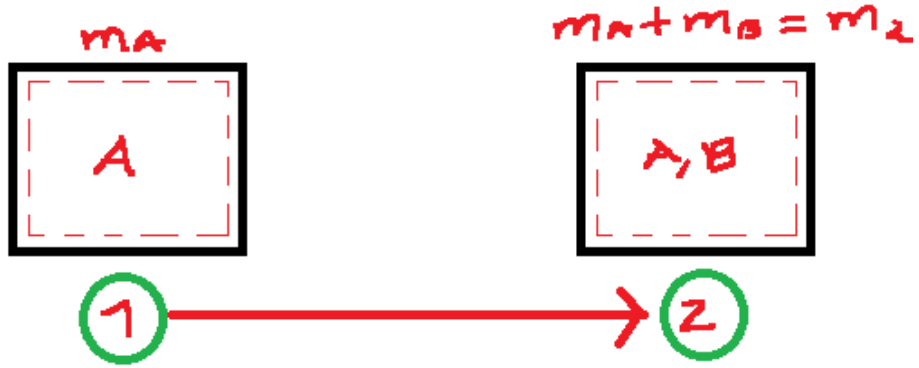
Mass change between state and 1 and 2



ΔH_{12}?

KJ

Δh_{12}?

$\frac{KJ}{kg}$

$$\Delta h_{12} = (m_{fA} h_A + m_{fB} h_B) - h_A$$

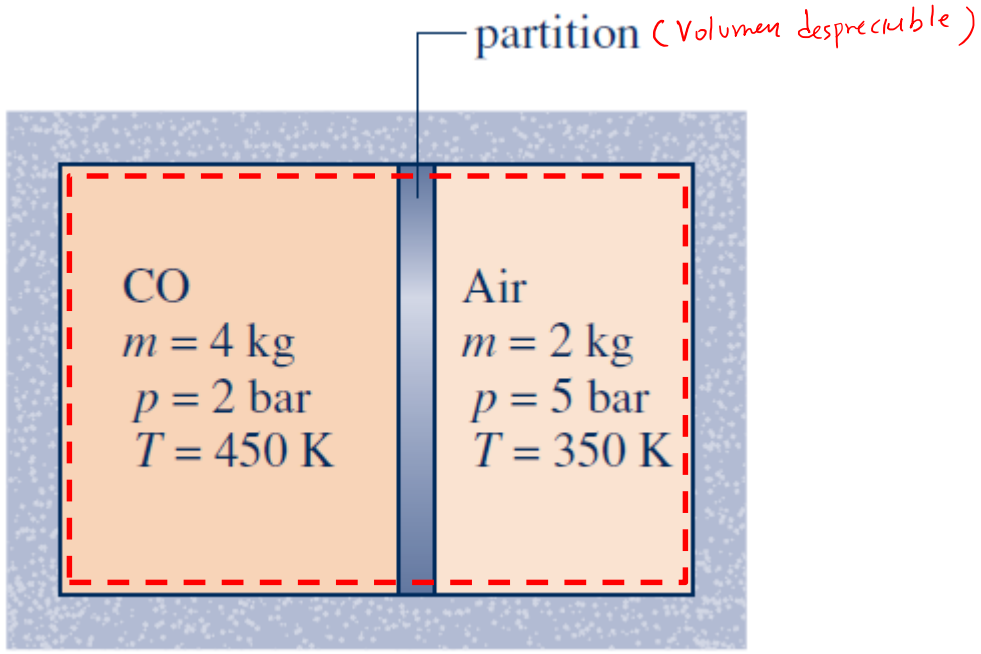
$$\Delta H_{12} = m_2 (m_f h_A + m_{fB} h_B) - m_A h_A$$

$m_A + m_B$ → in (2)

h can be calculated using Cp or tables.

EXAMPLE 8.2: ΔU_m [kJ] using tables_Mass change between state and 1 and 2

2 kg of air, initially at 5 bar, 350 K and 4 kg of carbon monoxide (CO) initially at 2 bar, 450 K are confined to opposite sides of a rigid container by a partition, as shown below. The partition is removed, and the final mixture reaches the equilibrium at 280K with the environment. The air and CO each behave as ideal gases it is possible to use tables A-22 and A23. Determine at equilibrium (a) the pressure, in Kpa, and (b) the total internal energy change: ΔU_m [kJ], considering the whole tank as system.



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Specifying both sides before mixing

CO side

$m_{A,1} = 4kg$

$P_{A,1} = 200kPa$

$T_{A,1} = 450K$

$$V_{A,1} = \frac{m_{A,1} \times R \times T_{A,1}}{P_{A,1}} = \frac{4 \times \frac{8.314}{28.01} \times 450}{200} = 2.6714\ m^3$$

Air side

$m_{B,1} = 2kg$

$P_{B,1} = 500kPa$

$T_{B,1} = 350K$

$$V_{B,1} = \frac{m_{A,1} \times R \times T_{A,1}}{P_{A,1}} = \frac{2 \times \frac{8.314}{28.97} \times 350}{500} = 0.4018\ m^3$$

a) After mixing, when the whole system reaches the equilibrium at 280K:

$$P_2 \times V_2 = n \times \bar{R} \times T$$

$$P_2 \times (V_{A,1} + V_{B,1}) = \left(\frac{m_{A,1}}{M_{CO}} + \frac{m_{B,1}}{M_{Air}}\right) \times \bar{R} \times T$$

$$P_2 \times (2.6714 + 0.4018) = \left(\frac{4}{28.01} + \frac{2}{28.97}\right) \times 8.314 \times 280$$

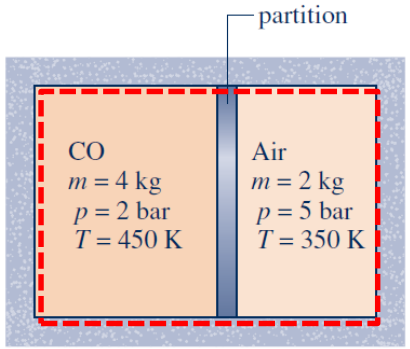
$$P_2 = 160.4691\ kPa$$

b) Internal energy change [kJ], in the whole system:

$$\Delta U_m = \left[n_{CO} \times \bar{u}_{co\_280K} + m_{Air} \times u_{air\_280K}\right] - \left[n_{CO} \times \bar{u}_{co\_450K} + m_{Air} \times u_{air\_350K}\right]$$

$$\Delta U_m = \left(\frac{4}{28.01} \times 5812 + 2 \times 199.75\right) - \left(\frac{4}{28.01} \times 9375 + 2 \times 250.02\right)$$

$$\Delta U_m = 1229.4893 - 1838.8476 = -609.3583 \text{ kJ}$$



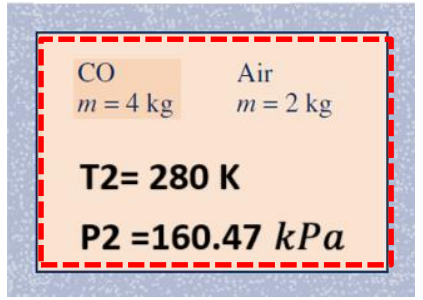


TABLE A-23

Ideal Gas Properties of Selected Gases

Enthalpy $\bar{h}(T)$ and internal ener

	Carbon Monoxide, CO ($\bar{h}_f^\circ$ = −110,530 kJ/kmol)		
T(K)	$\bar{h}$	$\bar{u}$	$\bar{s}^\circ$
280	8,140	5,812	195.173
450	13,116	9,375	209.593

TABLE A-22

Ideal Gas Properties of Air

T(K), h and u(kJ/kg), s° (kJ/kg · K)

T	h	u	s°
280	280.13	199.75	1.63279
350	350.49	250.02	1.85708

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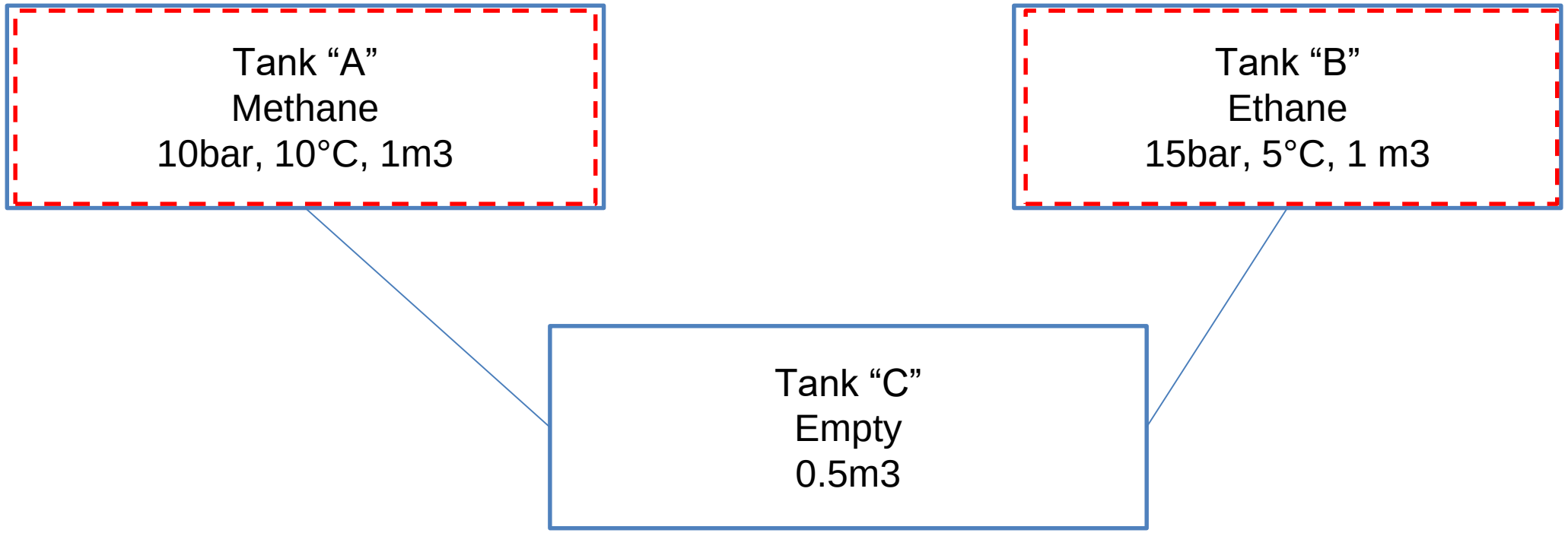
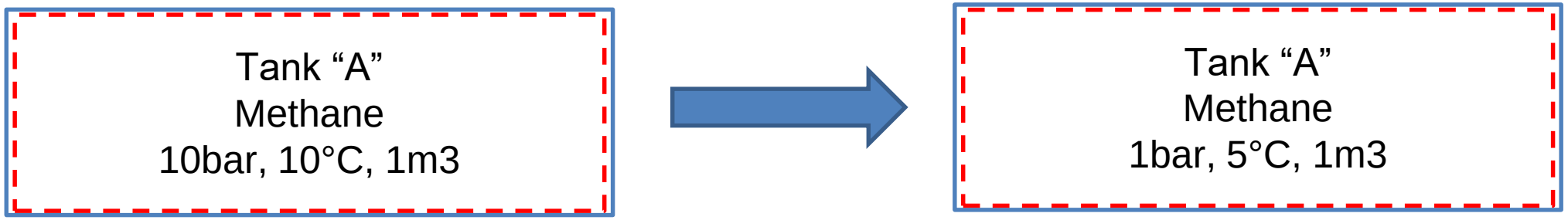
EXAMPLE 8.1:

Two rigid tanks “A” and “B” contain methane and ethane respectively at the conditions shown below. Methane valve is opened until the pressure and temperature inside the tank decreases to 1bar and 5°C, then, ethane valve is opened until its temperature drops to 0°C. The temperature in tank “C” is 20°C during the whole process and it is required that the ethane mol fraction be 20%. Find a) the change of internal energy in the tank “A” (kJ), b) the change in enthalpy in the tank “B” (kJ), c) the total mass (kg) and final pressure inside tank “C” (bar)

$\bar{c}_{p,methane} = 36\ kJ/kmolK$, $\bar{c}_{\ v,ethane} = 44.7 kJ/kmolK$



a) For tank “A”



Moles in “A”, before opening vale

$$n_1 = \frac{PV}{\bar{R}T} = \frac{(10 \times 100) \times 1}{8.314 \times (10 + 273.15)}$$

$$n_1 = 0.4248\ kmol$$

Moles in “A”, after opening vale

$$n_2 = \frac{PV}{\bar{R}T} = \frac{(1 \times 100) \times 1}{8.314 \times (5 + 273.15)}$$

$$n_2 = 0.0432\ kmol$$

Change of internal energy:

$$\Delta U = U_2 - U_1$$

$$\Delta U = n_2 u_2 - n_1 u_1$$

$$\Delta U = n_2 \bar{c}_v T_2 - n_1 \bar{c}_v T_1 = \bar{c}_v (n_2 T_2 - n_1 T_1)$$

$$\Delta U = (36 - 8.314) \times (0.0432 \times 278.15 - 0.4248 \times 283.15)$$

$$\Delta U = -2997.4536\ kJ$$

$\bar{c}_p = \bar{c}_v + \bar{R}$

$\bar{c}_v = \bar{c}_p - \bar{R}$

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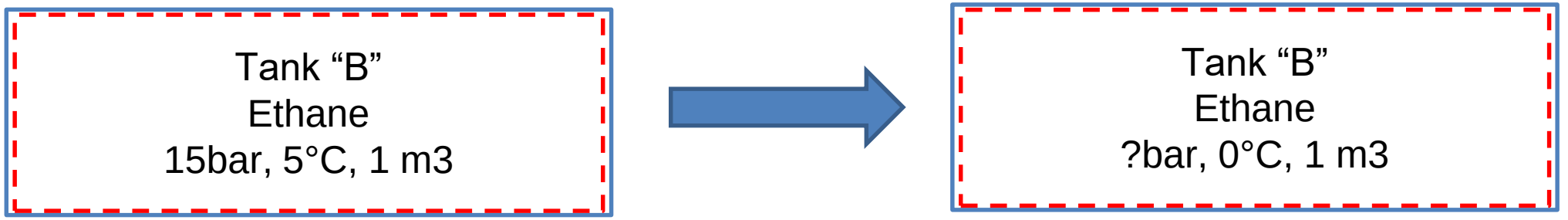
b) For tank “B”



Moles in “B”, before opening vale

$$n_1 = \frac{PV}{\bar{R}T} = \frac{(15 \times 100) \times 1}{8.314 \times (5 + 273.15)}$$

$$n_1 = 0.6486\ kmol$$

The amount of moles that went to tank “C”:

$$y_{ethane} = 0.2 = \frac{n_{E,C}}{n_{E,C} + (0.4248 - 0.0432)}$$

$$n_{E,C} = 0.0954\ kmol$$

Moles remaining of ethane in tank “B”:

$$n_2 = 0.6486 - 0.0954 = 0.5532\ kmol$$

Change of enthalpy

$$\Delta H = H_2 - H_1$$

$$\Delta H = n_2 h_2 - n_1 h_1 = n_2 \bar{c}_p T_2 - n_1 \bar{c}_p T_1 = \bar{c}_{p,ethane} (n_2 T_2 - n_1 T_1)$$

$$\bar{c}_p = \bar{c}_v + \bar{R}$$

$$\Delta H = (44.7 + 8.314) \times (0.5532 \times 273.15 - 0.6486 \times 278.15)$$

$$\boldsymbol{\Delta H} = -1553.3903\ kJ$$

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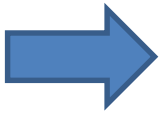
c) For tank “C”

Tank “C”
Empty



Tank “C”
$(0.4248 - 0.0432)$ = 0.3816 kmol Methane
0.0954kmol Ethane

Total mass:

$$m = n_1 M_1 + n_2 M_2$$

$$m = 0.3816 \times 16.04 + 0.0954 \times 30.07$$

$$m = 8.9895\ kg$$

Final pressure:

$$P_m = \frac{n_m \bar{R} T_m}{V_m} = \frac{(0.3816 + 0.0954) \times 8.314 \times (20 + 273.15)}{0.5}$$

$$P_m = 2325.1356\ kPa = 23.2514\ bar$$

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