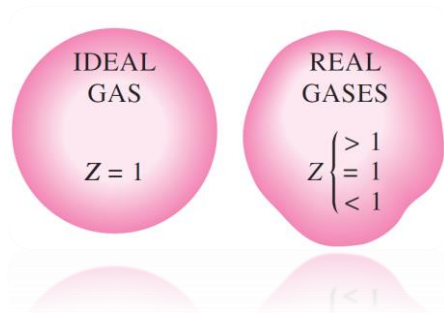


Thermodynamic 2026-1

NEDHER SÁNCHEZ RAMÍREZ

NAUDY LOPEZ

Ideal and Real Gases



Recomendaciones generales:

- Leer antes de clase y entender los conceptos o preguntar si es necesario.
- Practicar bastante:
 - *Ejercicios de clase
 - *Asesorías
 - *Ejercicios del libro resueltos y los de final de capítulo
- Los quizzes virtuales y gradescope úsenlos para aprender de forma honesta (se puede inferir).
- Tengan grupos de estudio y apóyense mutuamente.
- Crear sus propias notas estudio y formulario.
- Son 4 horas de clase en la semana (asistir y ser puntuales, perder alguna es atrasarse mucho)
- Cada hora de clase son 3 horas de estudio por fuera mínimo.
- En la ppt de la semana 1 están todos los links claves para el curso.

Instructores del curso y coordinador



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nsanchezr@utec.edu.pe



squispe@utec.edu.pe

Guidelines for virtual sessions:



Tiempo aproximado de la sesión 100 minutos



Silenciar su micrófono.



Realizar preguntas a través del chat, habrá un profesor y/o un asistente atento a las dudas planteadas



Levantar la mano si desea hacer uso del micrófono y exponer alguna duda



La grabación de la sesión se cargará a Canvas.



MARZO						
Lu.	Ma.	Mi.	Ju.	Vi.	Sá.	Do.
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30	31					

2
3
4
5
6

ABRIL						
Lu.	Ma.	Mi.	Ju.	Vi.	Sá.	Do.
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30			

MAYO						
Lu.	Ma.	Mi.	Ju.	Vi.	Sá.	Do.
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
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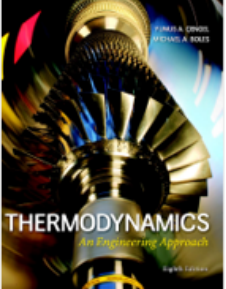
JUNIO						
Lu.	Ma.	Mi.	Ju.	Vi.	Sá.	Do.
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30					

JULIO						
Lu.	Ma.	Mi.	Ju.	Vi.	Sá.	Do.
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

15
16

Actividades	Inicio		Fin	
	Día	Fecha	Día	Fecha
Inicio de clases UTEC	Lunes 23 de marzo 2026			
Exámenes parciales	Lunes 11 de mayo 2026		Sábado 16 de mayo 2026	
Último día de clases	Sábado 4 de julio 2026			
Exámenes finales	Lunes 6 de julio 2026		Sábado 11 de julio 2026	
Exámenes de rezagados	Jueves 16 de julio 2026		Lunes 20 de julio 2026	

TEMAS SEMANA X SEMANA

Semana	Moran M.J. & Shapiro H.N. (2004). Fundamentos de la termodinámica Técnica. 2da edición (español). 	Cengel, Y.A. & Boles, M.A. (2015). Thermodynamics: An Engineering Approach. Eighth Edition (inglés). 
1	Capítulo 1: secciones 1.1 – 1.6 Ejercicios: 1.1 – 1.28	Capítulo 1: 1.3 – 1.9
2	Capítulo 3: secciones 3.1 – 3.2, 3.3.1 – 3.3.2, 3.3.6 Ejercicios: 3.1 – 3.26	Capítulo 3: 3.1 – 3.5 Capítulo 4: 4.1, 4.5
3	Capítulo 3: 3.3.5, 3.4 – 3.8 Ejercicios: 3.40 – 3.51	Capítulo 3: 3.6 – 3.7 Capítulo 4: 4.4
4	Capítulo 12: 12.1 – 12.4 Ejercicios: 12.1 – 12.6	Capítulo 13: 13.1 – 13.3
5	Capítulo 2: 2.1 – 2.6 Capítulo 3: ejemplos 3.2 – 3.4, 3.8 – 3.11	Capítulo 2: 2.1 – 2.5
6	Ejercicios: 2.1 – 2.10, 2.12 – 2.13, 2.21 – 2.40, 3.27 – 3.39, 3.52 – 3.58, 12.7 – 12.8, 12.10, 12.16 – 12.19	Capítulo 2: 2.6 Capítulo 4: 4.1 – 4.2
7	Capítulo 4: 4.1 – 4.4	Capítulo 5: 5.1 – 5.5
8	Ejercicios: 4.1 – 4.33	
9	Capítulo 5: 5.1 – 5.4 Ejercicios: 5.1 – 5.11, 5.14 – 5.33	Capítulo 6: 6.1 – 6.4, 6.6
10	Capítulo 5: 5.2, 5.6 – 5.7 Ejercicios: 5.14 – 5.33	Capítulo 6: 6.7 – 6.8, 6.10 – 6.11
11	Capítulo 6: 6.1 – 6.5 Ejercicios: 6.1 – 6.31	Capítulo 7: 7.1 – 7.10
12	Capítulo 6: 6.6 Ejercicios: 6.32 – 6.43	Capítulo 7: 7.13
13	Capítulo 6: 6.7 – 6.8	
14	Ejercicios: 6.44 – 6.56	Capítulo 7: 7.12

Importante: Las lecturas señaladas en este documento deben ser leídas antes de la semana indicada, en preparación para la tarea semanal de dicha semana. Ambos libros cubren aproximadamente el mismo contenido, pero con enfoques y niveles de profundidad distintos. Los estudiantes podrán alcanzar la comprensión del curso empleando solo una de las referencias. Sin embargo, se invita a contrastar semanalmente los temas en ambos textos, como método para alcanzar la comprensión máxima del contenido. Tener en cuenta que los subcapítulos indicados en esta guía corresponden a las ediciones de los libros especificadas.

▼ Semana 1	✓	+	⋮
Material de clase	✓		⋮
Actividades	✓		⋮
Material complementario	✓		⋮
📎 Cengel & Boles - Semana 1.pdf	✓		⋮
📎 Moran & Shapiro - Semana 1.pdf	✓		⋮

Horas de atención - Jhordy Manrique
(Dudas del curso-Tareas - notas)
LUNES 11-1 PM - Oficina P311

Horas de atención - Prof. Naudy Lopez
nlopez@utec.edu.pe
Previa coordinación con el profesor

Horas de atención - Jhordy Manrique
(Dudas del curso-Tareas - notas)
JUEVES 10-12 AM - Oficina P311

Horas de atención - Prof. Nedher Sánchez Ramírez
JUEVES: 9-12 pm....1-2pm - Oficina P311

Hora	Horas de asesoría 2026-1						Hora
	Lunes	Martes	Miércoles	Jueves	Viernes	Sábado	
7AM - 8AM		Clases		Clases			7AM-8AM
8AM - 9AM		Clases		Clases			8AM - 9AM
9AM - 10AM		Clases	Clases				9AM - 10AM
10AM - 11AM		Clases	Clases				10AM - 11AM
11AM - 12PM			Clases				11AM-12PM
12PM - 1PM			Clases				12PM-1PM
1PM - 2PM			Clases				1PM - 2PM
2PM - 3PM			Clases	CT: https://utec.zoom.us/j/97371427800			2PM - 3PM
3PM - 4PM	Clases					FR: https://utec.zoom.us/j/95953412067	3PM - 4PM
4PM - 5PM	Clases						4PM - 5PM
5PM - 6PM	Clases	Clases	Clases				5PM - 6PM
6PM - 7PM	Clases	Clases	Clases				6PM - 7PM
7PM - 8PM							7PM - 8PM
8PM - 9PM	ASESORÍA DE DUDAS: https://utec.zoom.us/j/97136492520	JG: https://utec.zoom.us/j/92301579107		RV: https://utec.zoom.us/j/92982722393			8PM - 9PM
9PM - 10PM							9PM - 10PM
RV: Rosario Vidal		Todas las sesiones serán grabadas.					
CT: Cristian Tobias		*Varia según disponibilidad					
FR: Franklin Rosas							
JG: Jose Gamero							

Link de WhatsApp de Termodinámica:

<https://chat.whatsapp.com/lu3b3wnWFuY2Zvamn89NNa>

Thermodynamics _ Evaluation

S_0 A
 S_{16} +

COMPONENTE	%
EXAMEN PARCIAL	20
EXAMEN FINAL	30
C	50
APROBACIÓN: 11/20	

EXÁMENES	FECHAS
EXAMEN PARCIAL	(MAYO 8)
EXAMEN FINAL	(JULIO 6)

EXAMEN 1: Todos los temas vistos hasta la semana 6.

EXAMEN 2: Todos los temas vistos hasta la semana 15

Thermodynamics _ Evaluation

COMPONENTE	%
EXAMEN PARCIAL	20
EXAMEN FINAL	30
C	50
APROBACIÓN: 11/20	

EVALUACIÓN CONTINUA (C) :

La evaluación continua vale un 50% de la nota final

-5 % corresponde a tareas virtuales ..Son de corrección automática

-10 % corresponde a **tareas individuales** (ejercicios que los alumnos hacen y suben a la plataforma gradescope)Ver rúbrica

-17 % corresponde a una **tarea grupal** (proyecto).....Ver rúbrica

-18% restante corresponde **actividades en aula** (quizes de 20 min aprox)..Ver rúbrica

Thermodynamics _ Evaluation

SEMANA 1	TAREA VIRTUAL	SE CALIFICA AUTOMATICO NO NECESITA PROCEDIMIENTO
SEMANA 2	TAREA VIRTUAL	
SEMANA 3	TAREA VIRTUAL	
		TAREA INDIVIDUAL: Esta parte incluye subir un archivo con la solución
SEMANA 4	TAREA INDIVIDUAL 1 + EVALUACION_AULA1	EVALUACION_AULA: SEMANAS 1-2
SEMANA 5	TAREA VIRTUAL+ EVALUACION_AULA2	EVALUACION_AULA: SEMANAS 1-3
SEMANA 6	TAREA VIRTUAL+TAREA INDIVIDUAL 2	
SEMANA 7	TAREA VIRTUAL+ EVALUACION_AULA3	EVALUACION_AULA: SEMANAS 1-6
SEMANA 8	PARCIAL	
	EVALUACION_AULA4+TAREA INDIVIDUAL 3 +Tarea grupal 1	
SEMANA 9		EVALUACION_AULA: SEMANAS 1-8
SEMANA 10	TAREA VIRTUAL	
SEMANA 11	TAREA VIRTUAL	
SEMANA 12	TAREA VIRTUAL+ EVALUACION_AULA5	EVALUACION_AULA: SEMANAS 1-11
SEMANA 13	TAREA VIRTUAL+TAREA INDIVIDUAL 4 + Tarea grupal 2	
SEMANA 14	TAREA VIRTUAL+EVALUACION_AULA6	EVALUACION_AULA : SEMANAS 1-13
SEMANA 15		
SEMANA 16	FINAL	

COMPONENTE	%
PARCIAL	20
FINAL	30
C	50
APROBACIÓN: 11/20	

***LAS TAREAS VIRTUALES O LOS GRADESCOPE (TAREA INDIVIDUAL) se abren lunes 2pm y se cierran sábado media noche. La tarea virtual se presenta hasta un máximo de 3 veces.**

EVALUACION_AULA: es un quiz de 20 min aprox. en aula.

Thermodynamics _ Evaluation

EVALUACIÓN CONTINUA (C) :

COMPONENT	
E	%
C	50

25 %

- 3 actividades en aula → 9 %
- 2 tareas individuales → 5 %
- ½ tareas virtuales → 2.5 %
- 1ra entrega tarea grupal → 8.5 %

25 %

- 3 actividades en aula → 9 %
- 2 tareas individuales → 5 %
- ½ tareas virtuales → 2.5 %
- 2da entrega tarea grupal → 8.5 %

Se subirá al EDU En dos
bloques de 25% c/v

Thermodynamics _ Evaluation

Tarea grupal (proyecto).....

La actividad consta del análisis de una sistema, dicho análisis puede requerir elementos de programación.

Fechas de entrega del laboratorio:

1ra entrega - 18/05/2026 a las 9 am (jueves de la semana 9)

2da entrega - 15/06/2026 a las 9am (lunes de la semana 13)

Este ciclo no hay co-evaluación !!!

Link con toda la información de la tarea grupal :

<https://docs.google.com/spreadsheets/d/1DMrZjoEaRIHfZPbrFa5q84ha0ElWJQS1zmyLanvjGcw/edit?gid=0#gid=0>

Thermodynamics _ Evaluation

Link con las rubricas de actividad de aula y tarea individual:

<https://docs.google.com/spreadsheets/d/1iLMFoTAZiUBYuD8fGZFW4d0pF0-UBt8f/edit?gid=85594443#gid=85594443>

Opening thoughts...

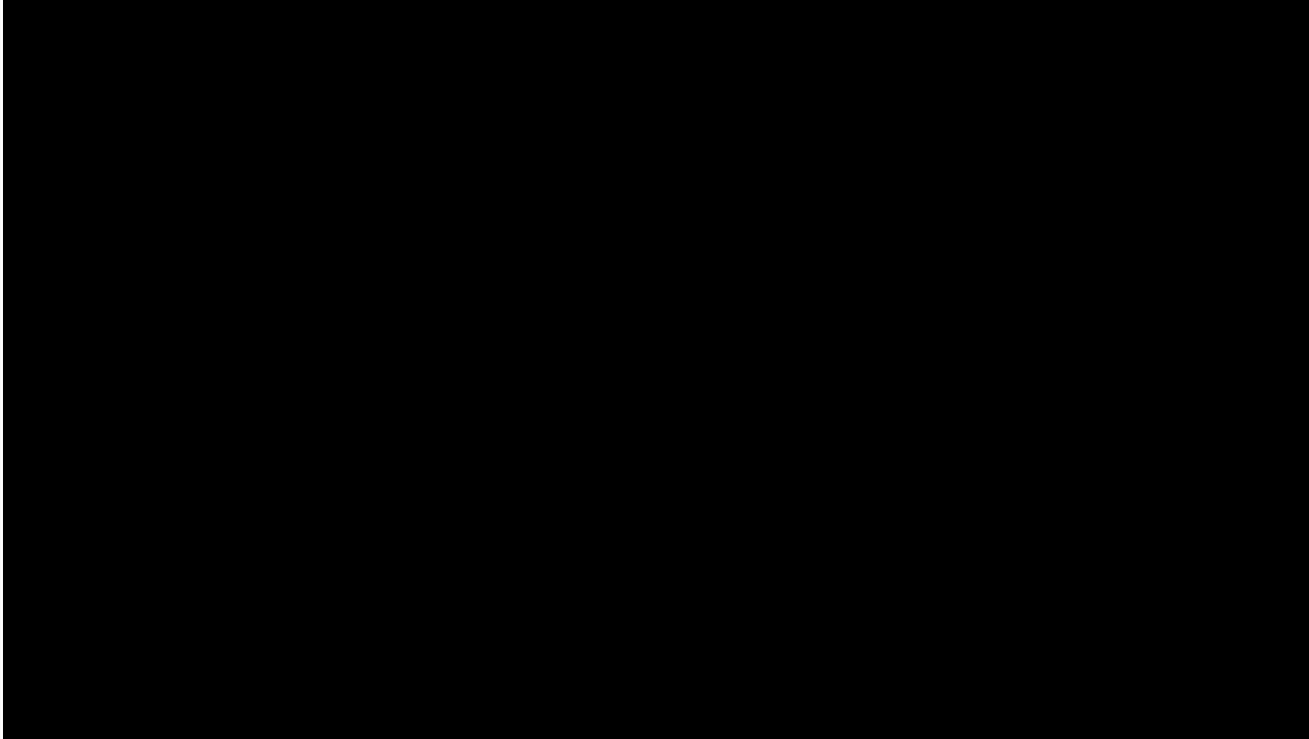
Have you ever...



$$PV = mRT$$
$$\frac{P}{RT} = \frac{m}{V} = \rho$$

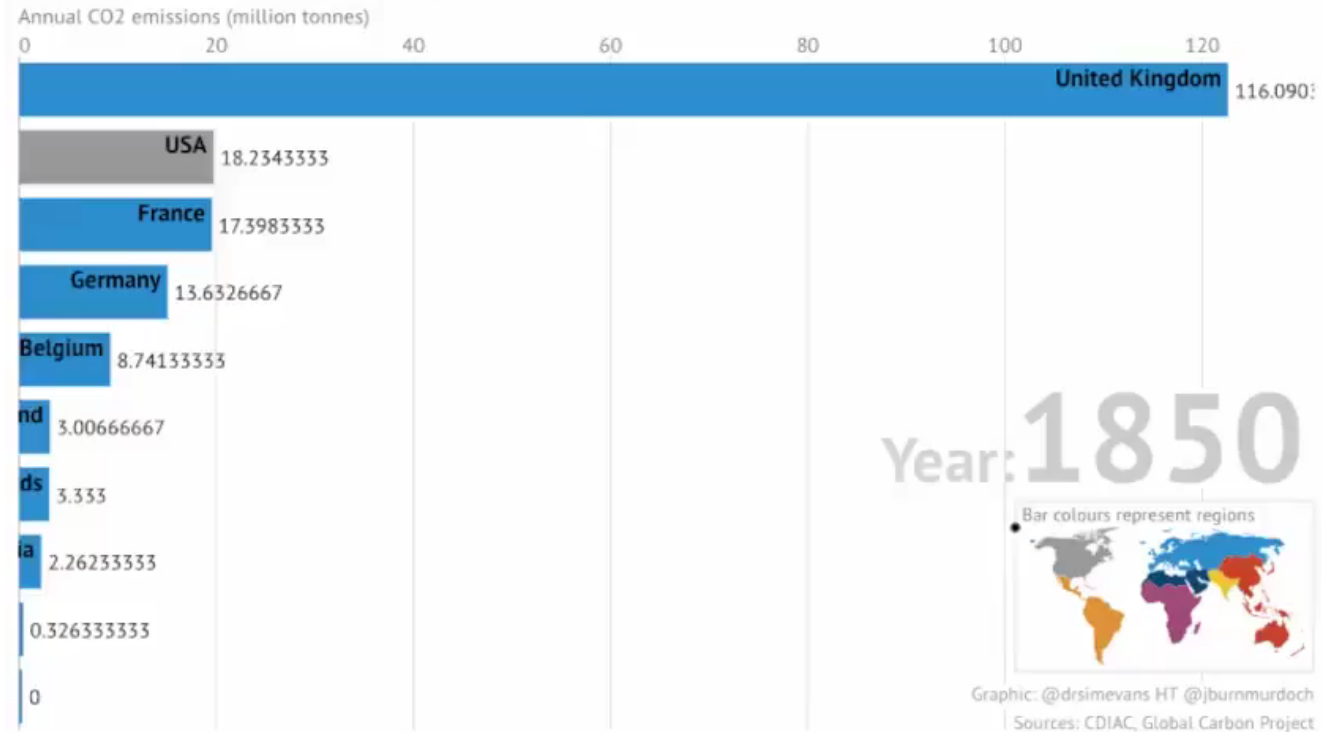
These are examples of **gases** !

Gases



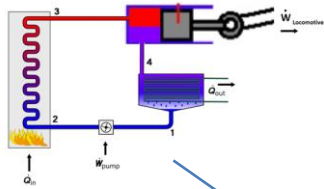
What this imply?

The countries with the largest CO2 emissions from 1850 to 2017



What tools do we need to understand these cycles?

Power cycle_ Rankine cycle (ideal)

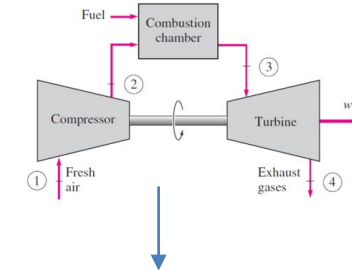


Refrigeration cycle_ Vapor-Compression Refrigeration Cycle

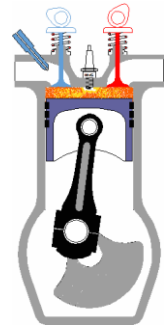


Pure substance: Water, refrigerants...

Power cycle_ Brayton cycle (ideal)



Pure substance: Ideal gas



1. Calculation of properties

Tables: Like those that are tabulated for some pure substances in the book appendices. **Week 2**

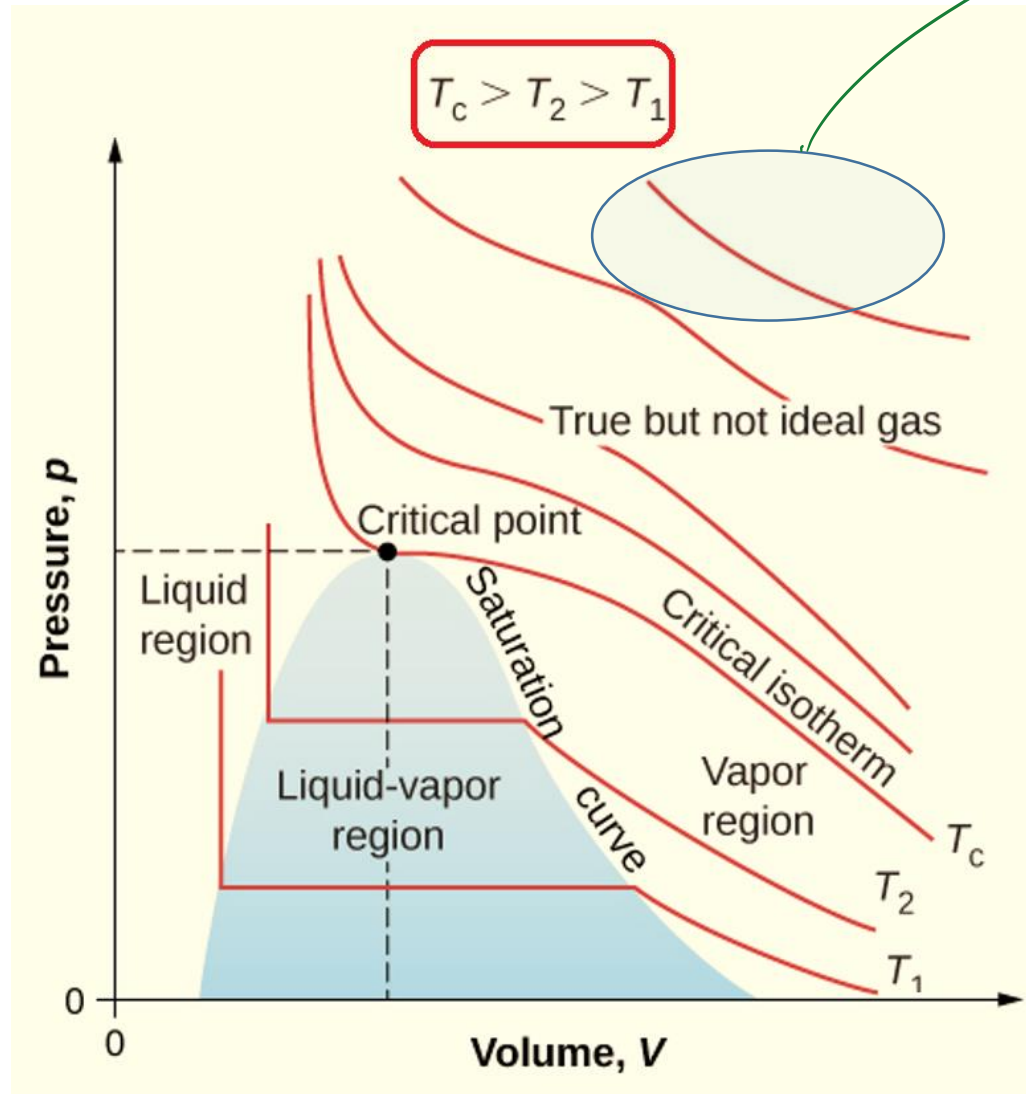
Models: As the ideal gas equation, or other relationships or approximations that allow calculating properties of liquids, solids or gases that are not tabulated. **Weeks 3-4**

2. Law of Thermodynamics

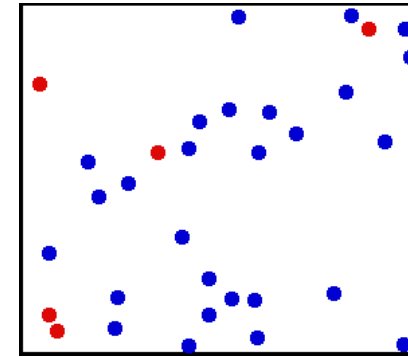
First Law of Thermodynamics: Energy Balance

Second Law of Thermodynamics: Entropy Balance

Ideal Gases



In this region the so-called **ideal gases** have some characteristics, allowing them to be described by a simple model based on macroscopic properties.



"Many substances that we treat as ideal gases such as air, CO₂, among others behave like ideal gases at ambient pressure and temperature."

Properties of Ideal Gases

You can predict the behavior of gases based on the following properties:

Pressure

Volume

Amount (moles)

Temperature

Amount (g-mol)

We can convert measured mass (in g) to the number of moles, n , using.....

Molar mass!

$25 \text{ g H}_2\text{O} \rightarrow \text{mol}?$

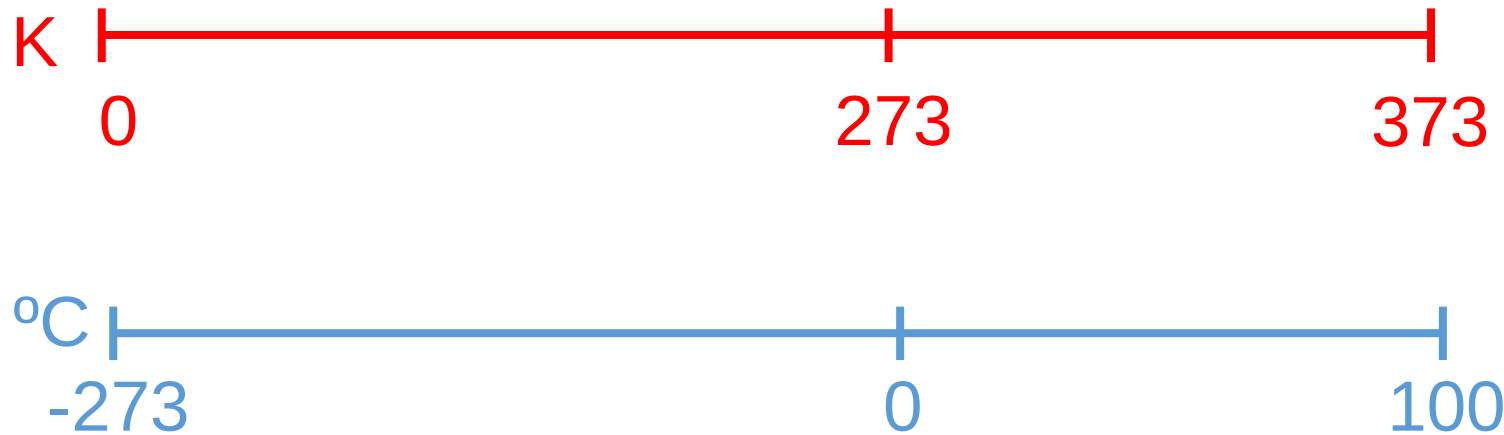


$$n = \frac{m}{M}$$

$$n = \frac{25 \text{ g}}{18 \frac{\text{g}}{\text{mol}}} = 1.39 \text{ mol H}_2\text{O}$$

Temperature

- Always use absolute temperature (Kelvin) when working with gases.



How do they all relate?

Now that we understand the factors that affect the behavior of gases, we will study how those factors interact.

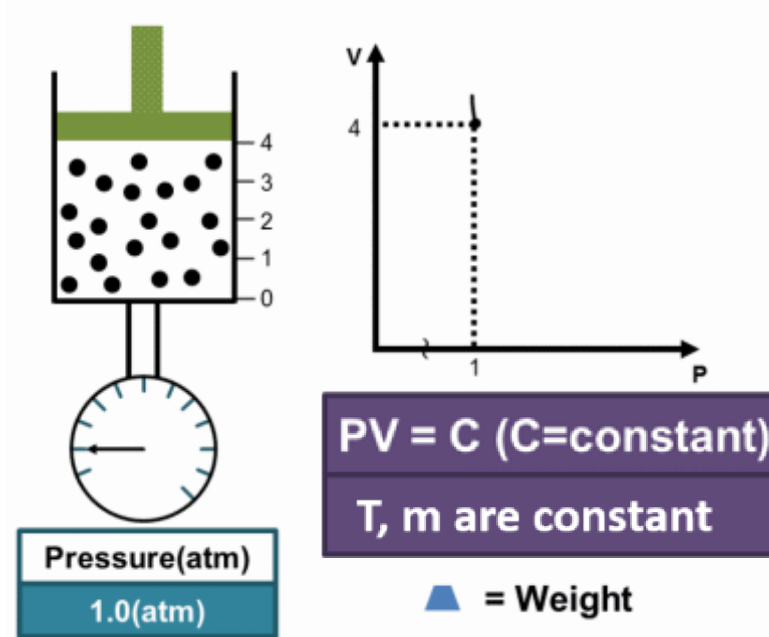
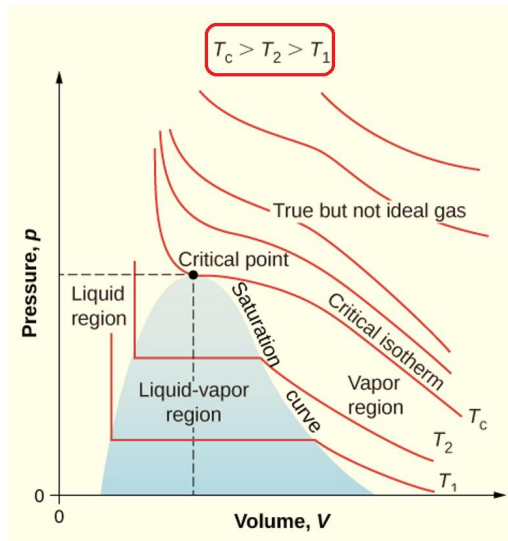
Recall

Boyle's Law	Charles' Law	Avogadro's Law
$V \propto 1/P$	$V \propto T \text{ (Kelvin)}$	$V \propto n$
Constant T, n	Constant P, n	Constant T, P

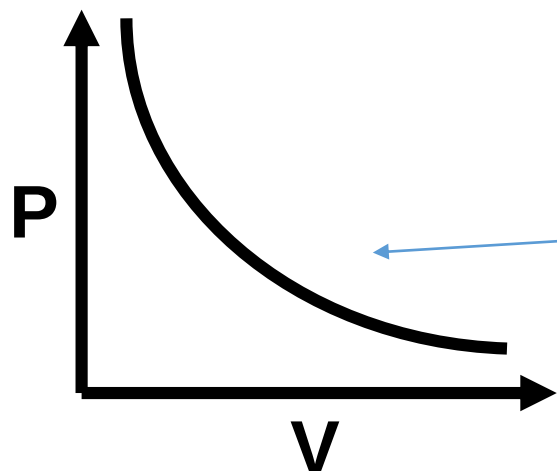
So $V \propto 1/P \times n \times T$

$$PV = n\bar{R}T$$

Boyle's Law _ P vs V



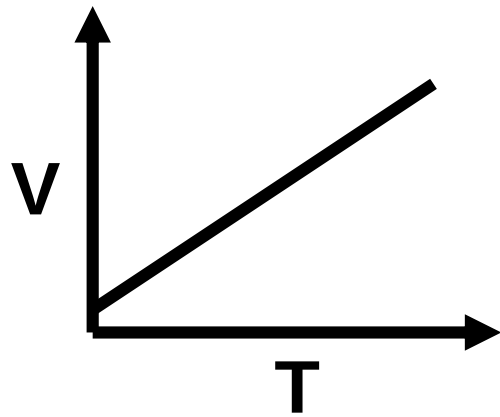
<https://unihisnordfrees.tumblr.com/post/146462022830/boyles-lesser-known-law-the-pressure-of-the-air>



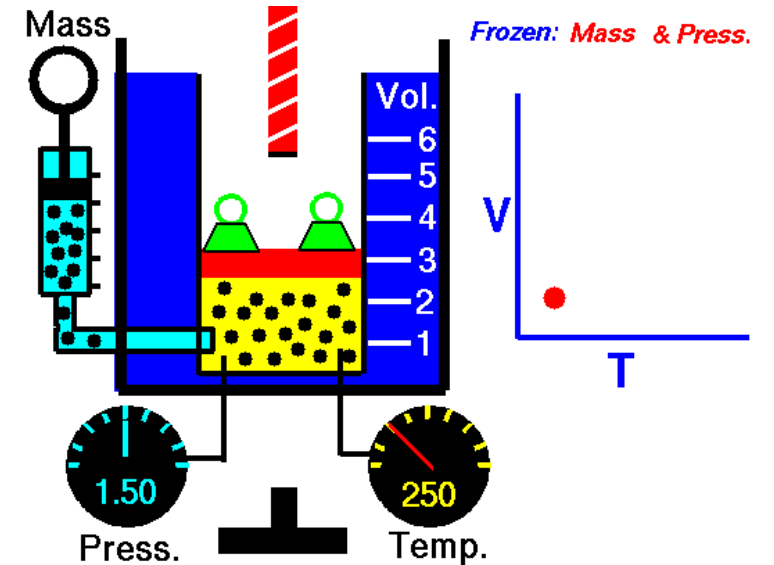
$$P_1 V_1 = P_2 V_2 = C$$

Charles' Law

This lesson introduces Charles' Law, which describes the relationship between volume and temperature of gases.



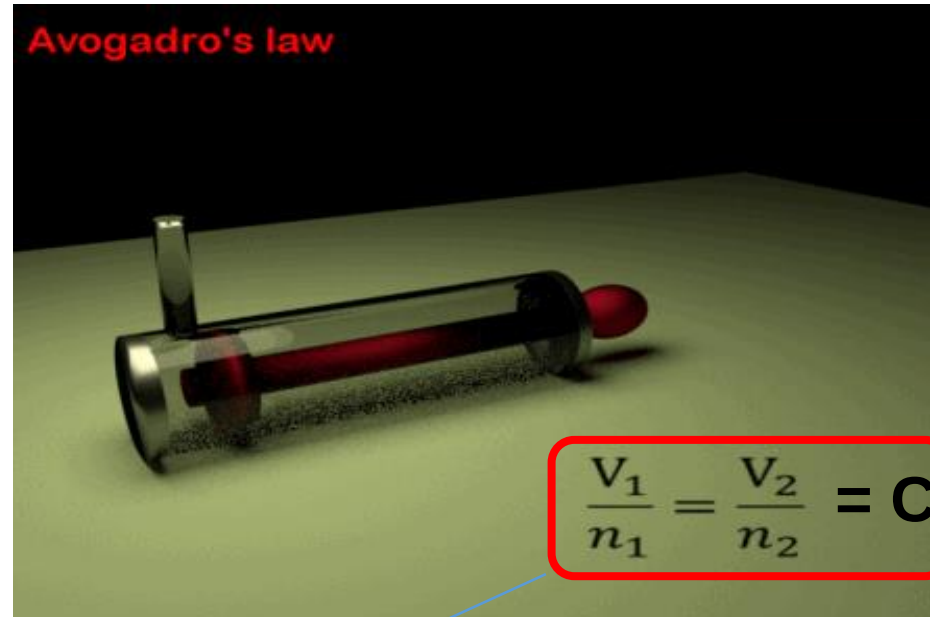
$$\frac{V_1}{T_1} = \frac{V_2}{T_2} = C$$



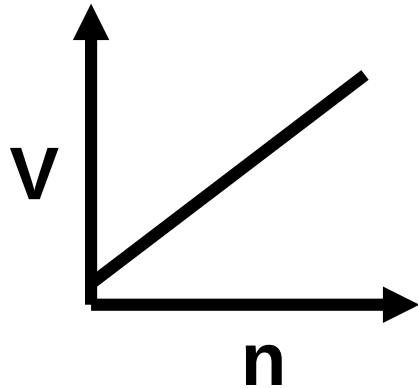
P, m are constant
 $V = C * T$

Avogadro's Law

- Equal volumes of gases contain equal numbers of moles at constant temp & pressure true for any ideal gas.



P, T are constant
 $V = C * n$



Recall

Boyle's Law	Charles' Law	Avogadro's Law
$V \propto 1/P$	$V \propto T \text{ (Kelvin)}$	$V \propto n$
Constant T, n	Constant P, n	Constant T, P

So $V \propto 1/P \times n \times T$

$$PV = n\bar{R}T$$

Ideal Gas Law

- To turn a proportionality into an equation, insert a constant:

$$PV = n\bar{R}T$$

Where “ $\bar{R}$ ” is the universal gas constant. v [m³]

$$n = m/M \text{ (mass/molar mass) } \dots PV/m = (\bar{R}/M)T$$

$$Pv = R T, \quad v \text{ [m}^3\text{/kg] } \dots \text{specific volume ; } “\bar{R}/M” = R_g = R = \text{gas constant}$$

Substance	$\bar{R}/M$, kJ/kg·K
Air	0.2870
Helium	2.0769
Argon	0.2081
Nitrogen	0.2968

↓
STATE EQUATION!

STATE POSTULATE FOR A PURE SUBSTANCE:

- Given Prop. 1 and Prop. 2 which are intensive and independent:

$$\text{Prop.3} = f(\text{Prop.1} + \text{Prop.2})$$

Ideal Gas Law

- Alternatively :

$$PV = n\bar{R}T, \quad V \text{ [m}^3\text{]}$$

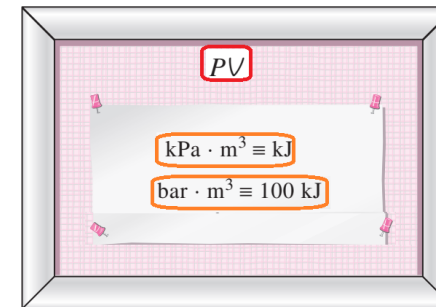
$$P\bar{v} = \bar{R}T, \quad \bar{v} \text{ [m}^3\text{/mol]} = \frac{V}{n}$$

“STATE EQUATION”

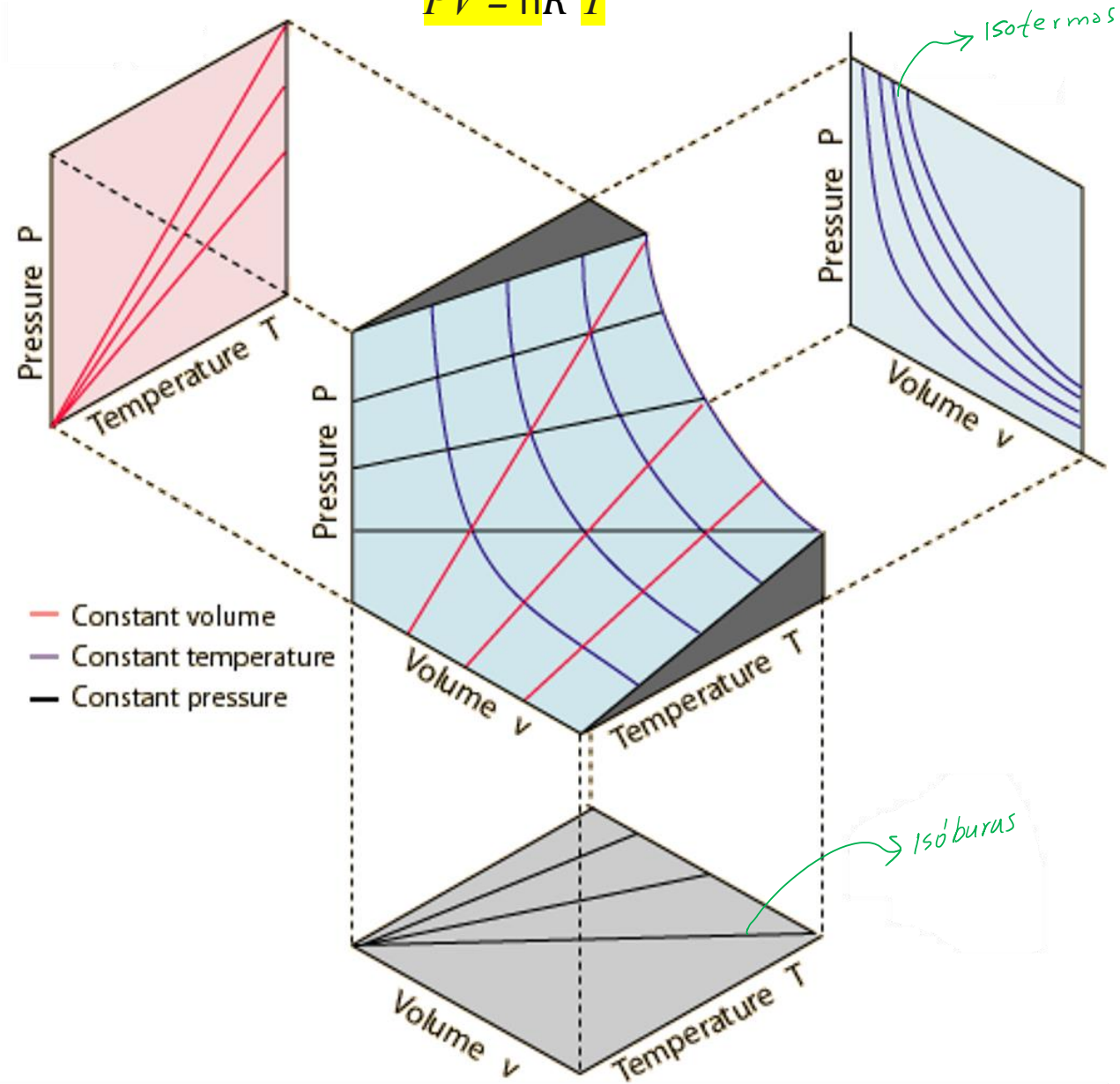
Ideal Gas Law

- The units of $\bar{R}$ depend on the units used for P, T, and V

$$\begin{array}{l} \bar{R} \\ R_U \end{array} = \left\{ \begin{array}{l} 0.0831447 \text{ bar} \cdot \text{m}^3 / \text{kmol} \cdot \text{K} \\ 8.31447 \text{ kPa} \cdot \text{m}^3 / \text{kmol} \cdot \text{K} \\ 8.31447 \text{ kJ} / \text{kmol} \cdot \text{K} \\ 0.082057 \text{ L} \cdot \text{atm} \cdot \text{K}^{-1} \cdot \text{mol}^{-1} \\ 8.205745 \times 10^{-5} \text{ m}^3 \cdot \text{atm} \cdot \text{K}^{-1} \cdot \text{mol}^{-1} \\ 8.314472 \text{ L} \cdot \text{kPa} \cdot \text{K}^{-1} \cdot \text{mol}^{-1} \\ 8.314472 \text{ m}^3 \cdot \text{Pa} \cdot \text{K}^{-1} \cdot \text{mol}^{-1} \\ 82.05745 \text{ cm}^3 \cdot \text{atm} \cdot \text{K}^{-1} \cdot \text{mol}^{-1} \\ 8.314472 \text{ J} \cdot \text{K}^{-1} \cdot \text{mol}^{-1} \end{array} \right.$$



$$PV = n\bar{R}T$$



EXAMPLE 1.4

Oxygen (Ideal gas) undergoes isothermal expansion, which increases its volume by 4.20 dm^3 ($V_2 = V_1 + 4.20$). The final pressure and volume of the gas are 2.50 bar and 5.20 dm^3 , respectively. Calculate the original pressure (P_1) of the gas in (a) bar, (b) kPa.

$$P_2 = 2.50 \text{ bar}$$

$$V_2 = 5.20 \text{ dm}^3$$

$$V_2 = V_1 + 4.20 \text{ dm}^3 \Rightarrow \textcircled{V_1} = 5.20 - 4.20 = 1 \text{ dm}^3$$

$P_1 ?$

(A) $\textcircled{P_1} V_1 = n \bar{R} T_1$

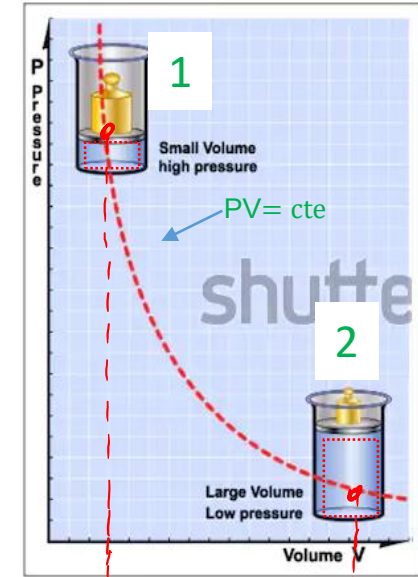
(B) $P_1 V_1 = n \bar{R} T_1$

$$\textcircled{P_1} V_1 = P_2 V_2$$

$$P_2 V_2 = n \bar{R} T_2$$

$T_1 \text{ (isotermico)}$

Boyle



www.shutterstock.com · 313381604

$$V_1 + 4.20$$

EXAMPLE 1.4

Oxygen (Ideal gas) undergoes isothermal expansion, which increases its volume by 4.20 dm³. The final pressure and volume of the gas are 2.50 bar and 5.20 dm³, respectively. Calculate the original pressure (P_1) of the gas in (a) bar, (b) kPa.

↓
 P_1

↓
 V_2

a) Boyle's law:

$$P_1 V_1 = P_2 V_2$$

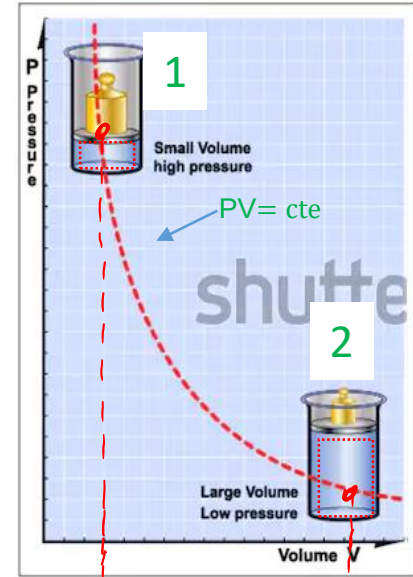
$$P_1 \times (5.20 - 4.20) = 2.50 \times 5.20$$

$$P_1 = 13.0 \text{ bar}$$

b) Converting bar to kPa:

$$P_1 = 13.0 \text{ bar} \times \frac{100 \text{ kPa}}{1.0 \text{ bar}} = 1300 \text{ kPa}$$

Boyle

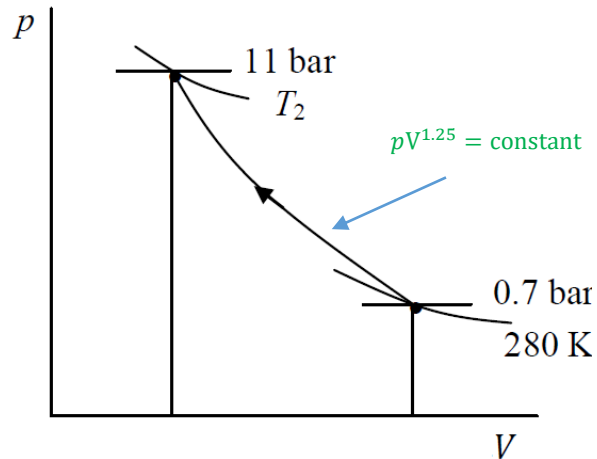
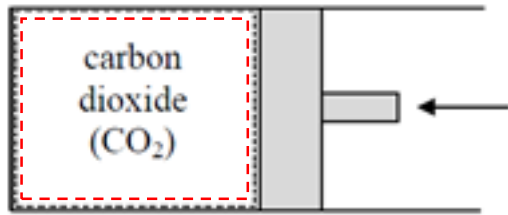


↓
 V_1

↓
 V_1
+
4.20

Example:

2.1. Carbon dioxide (CO_2) is slowly compressed in a piston-cylinder assembly from $p_1 = 0.7\text{bar}$, $T_1 = 280\text{K}$ to $p_2 = 11\text{bar}$. The initial volume is 0.400 m^3 . The process is described by $pV^{1.25} = \text{constant}$. Assuming ideal gas determine V_2 (m^3).



Process: $p_1 V_1^{1.25} = p_2 V_2^{1.25}$

$$0.7 \times 0.400^{1.25} = 11 \times V_2^{1.25}$$

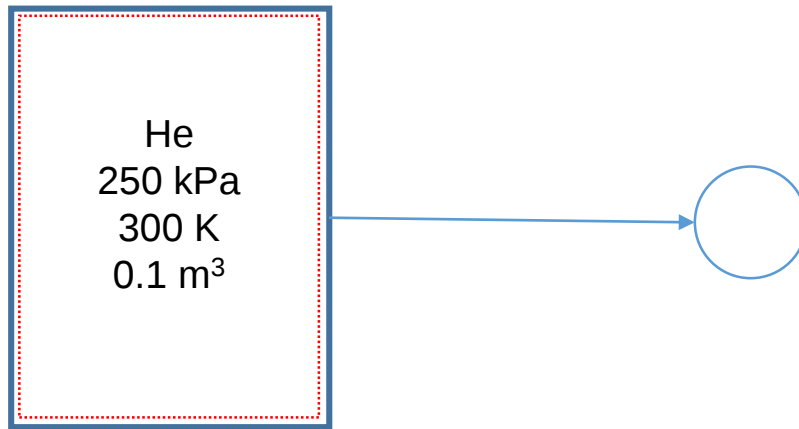
$$V_2^{1.25} = \frac{0.7 \times 0.400^{1.25}}{11} = 0.02024$$

$$V_2 = 0.04416\text{m}^3$$

Example

2.1 A rigid steel tank with a volume of 0.1 m^3 contains helium at 250 kPa and 300 K (**State 1**). The helium is used to fill a balloon until the tank pressure drops to 125 kPa (**State 2**), at which point the flow stops. Assuming an isothermal process, determine the amount of helium (in kmol) transferred to the balloon. Treat helium as an ideal gas.

INITIAL STATE

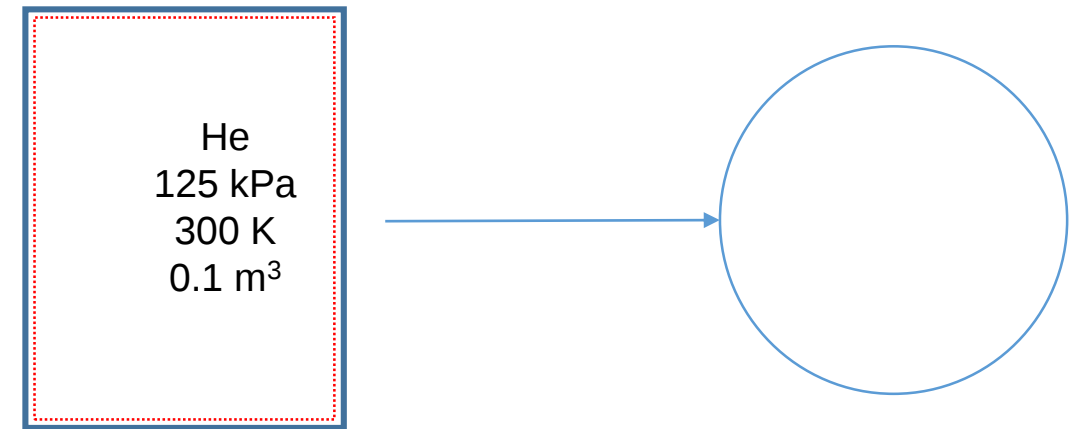


$$P_1 V_1 = n_1 \bar{R} T_1$$

$$250 \text{ kPa} \times 0.1 \text{ m}^3 = n_1 \times 8.314 \frac{\text{kPa} \cdot \text{m}^3}{\text{kmol} \cdot \text{K}} \times 300 \text{ K}$$

$$n_1 = 0.01002 \text{ kmol}$$

FINAL STATE



$$P_2 V_2 = n_2 \bar{R} T_2$$

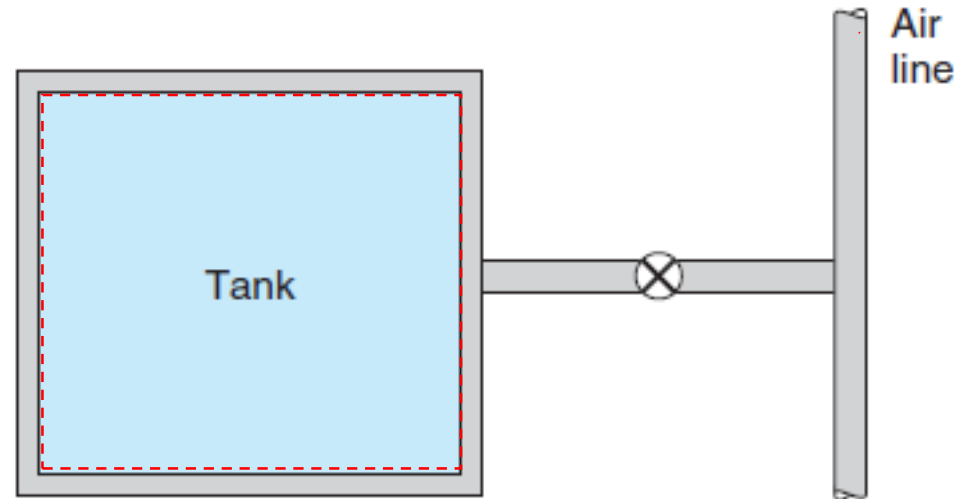
$$125 \text{ kPa} \times 0.1 \text{ m}^3 = n_2 \times 8.314 \frac{\text{kPa} \cdot \text{m}^3}{\text{kmol} \cdot \text{K}} \times 300 \text{ K}$$

$$n_2 = 0.00501 \text{ kmol}$$

Therefore, 0.00501 kmol has been transferred to the balloon

EXAMPLE 4.3

A 1-m³ rigid tank with air at 1 MPa and 400 K is connected to an air line as shown below. The valve is opened and air flows into the tank until the pressure reaches 5 MPa, at which point the valve is closed and the temperature inside is 450 K. a. What is the mass [kg] of air in the tank before and after the process? b. The tank eventually cools to room temperature, 300 K. What is the pressure inside the tank then [kPa]?



The air **before** opening the valve is (state 1):

$$m_1 = \frac{P_1 V}{RT_1} = \frac{1000 \times 1}{\frac{8.314}{28.97} \times 400} = 8.7112 \text{ kg}$$

The air **after** opening the valve and letting the air from the line enters (state 2):

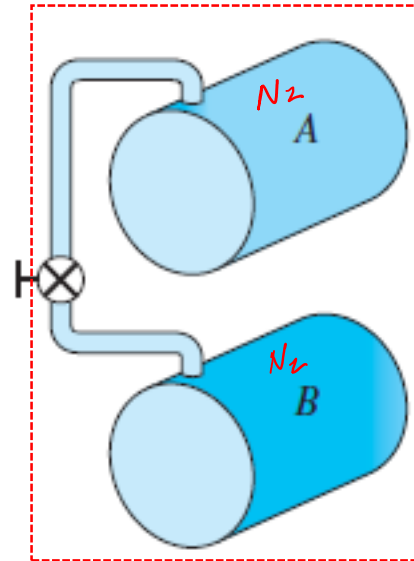
$$m_2 = \frac{P_2 V}{RT_2} = \frac{5000 \times 1}{\frac{8.314}{28.97} \times 450} = 38.7165 \text{ kg}$$

After cooling to 300 k (state 3):

$$P = \frac{m_2 R T_3}{V} = \frac{38.7165 \times \frac{8.314}{28.97} \times 300}{1} = 3333.3343 \text{ kPa}$$

EXAMPLE 5.3

A 1-m^3 rigid tank has nitrogen at 100 kPa , 300 K and connected by a valve to another rigid tank of 0.5 m^3 with nitrogen at 250 kPa , 400 K . The valve is opened, and the two tanks come to a uniform state at 325 K . What is the final pressure?



Nitrogen mass in tank A:

$$m = \frac{PV}{RT} = \frac{100 \times 1}{\frac{8.314}{28.01} \times 300} = 1.1230 \text{ kg}$$

Nitrogen mass in tank B:

$$m = \frac{PV}{RT} = \frac{250 \times 0.5}{\frac{8.314}{28.01} \times 400} = 1.0528 \text{ kg}$$

When the valve is opened, the final pressure is:

$$P = \frac{m_T RT}{V} = \frac{(1.1230 + 1.0528) \times \frac{8.314}{28.01} \times 325}{1 + 0.5} = 139.9291 \text{ kPa}$$

Example:

7.1. 2 moles of an ideal monoatomic gas have an initial pressure $p_1 = 3\text{atm}$ and an initial volume $V_1 = 4\text{L}$. The gas is taken through the following quasi-static cycle:

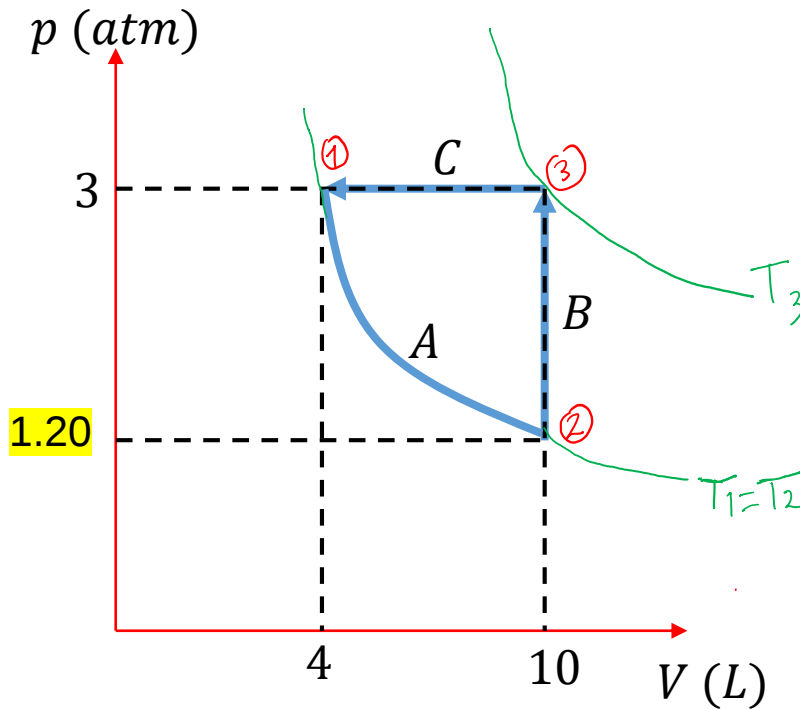
A.- It is expanded isothermally until it has a volume $V_2 = 10\text{L}$.

B.- It is then heated at constant volume until it has a pressure $p_3 = 3\text{atm}$

C.- It is then cooled at constant pressure until it is back to its initial state.

(a) Show this cycle on a PV diagram. (b) Find the temperatures T_1 , T_2 , T_3

	0.0831447 bar · m ³ /kmol · K
	8.31447 kPa · m ³ /kmol · K
	8.31447 kJ/kmol · K
$\overline{R}$	0.082057 L · atm · K ⁻¹ · mol ⁻¹
	$8.205745 \times 10^{-5} \text{ m}^3 \cdot \text{atm} \cdot \text{K}^{-1} \cdot \text{mol}^{-1}$
	8.314472 L · kPa · K ⁻¹ · mol ⁻¹
	8.314472 m ³ · Pa · K ⁻¹ · mol ⁻¹
R_U	82.05745 cm ³ · atm · K ⁻¹ · mol ⁻¹
	8.314472 J · K ⁻¹ · mol ⁻¹



State 1:

$$p_1 V_1 = nRT_1$$

$$3 \times 4 = 2 \times 0.082057 \times T_1 \rightarrow T_1 = 73.1199 \text{ K}$$

State 2: Process A isothermal

$$T_1 = T_2 \rightarrow T_2 = 73.1199 \text{ K}$$

$$P_1 V_1 = P_2 V_2 \dots P_2 = P_1 V_1 / V_2 = 3 \times 4 / 10 = 1.2 \text{ atm}$$

State 3:

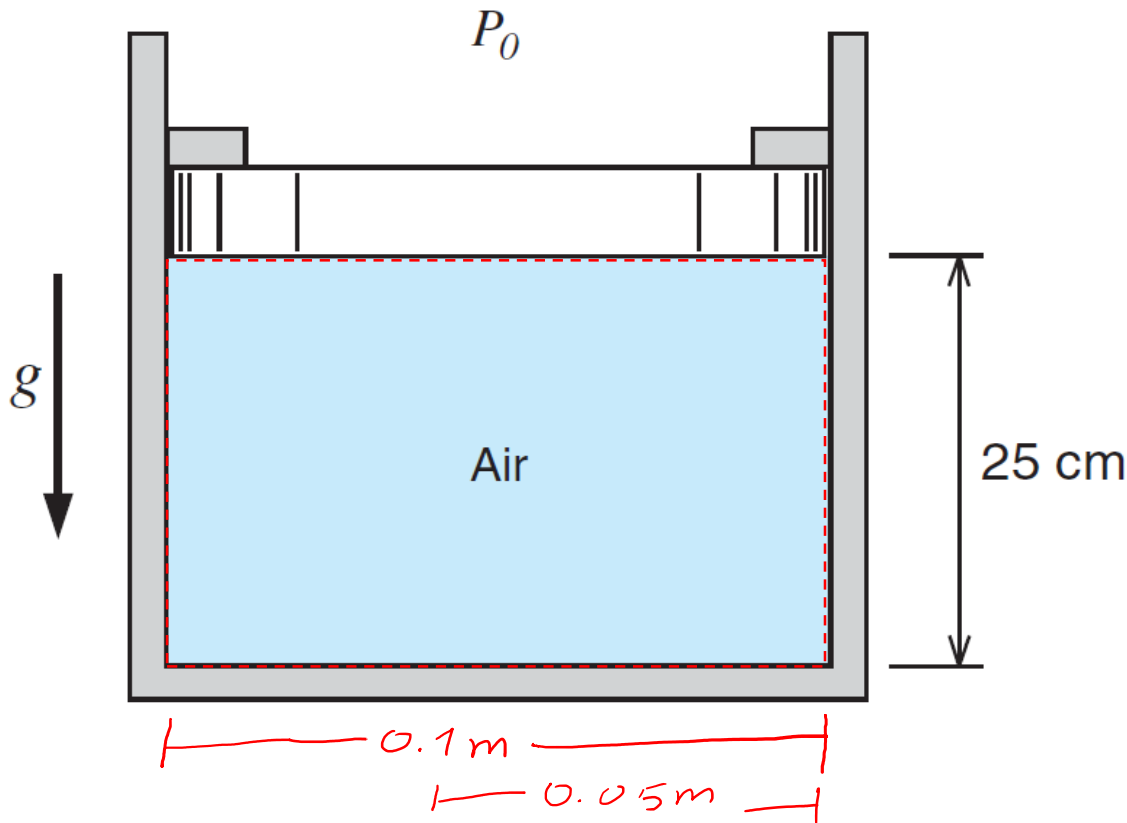
$$p_3 V_3 = nRT_3$$

$$3 \times 10 = 2 \times 0.082057 \times T_3 \rightarrow T_3 = 182.7998 \text{ K}$$

EXAMPLE 3.9

A piston/cylinder (without friction) arrangement, shown below, contains air at 250 kPa and 300°C. The 50-kg piston has a diameter of 0.1 m and initially pushes against the stops. The atmosphere is at 100 kPa (P_0) and $T_{\text{ambient}} = 20^\circ\text{C}$. The cylinder now slowly cools as heat is transferred to the ambient surroundings. Assume air as an ideal gas.

- At what temperature does the piston **just** begin to move down? [K]
- How far has the piston dropped when the temperature reaches ambient? [m]
- Show the process in a **P [kPa]–V [L]** and a **T [K]–V [L]** diagram.



$$M(\text{air}) = 28.97 \frac{\text{g}}{\text{mol}} = 28.97 \frac{\text{kg}}{\text{kmol}}$$

state 1 $\left\{ \begin{array}{l} T_1, P_1 \\ V \rightarrow \text{geomet} \\ m \rightarrow E_c \end{array} \right.$

state 2 $\left\{ \begin{array}{l} V_2 = V_1, m \\ P_2 \rightarrow m \cdot F \\ T_2 \rightarrow E_c \end{array} \right.$

state 3 $\left\{ \begin{array}{l} T_3, m \\ P_3 = P_2 \\ V_3 \rightarrow E_c \\ \downarrow \\ A \cdot h_3 = V_3 \end{array} \right.$

With the initial conditions **(STATE 1)** we can calculate mass of air inside the cylinder:

$$m_{air} = \frac{P_1 V_1}{R T_1} = \frac{250 \times (\pi \times 0.05^2 \times 0.25)}{\frac{8.314}{28.97} \times (300 + 273.15)} = 2.984 \times 10^{-3} \text{ kg}$$

$$\bar{R} [=] 8.314 \frac{\text{KPa} \cdot \text{m}^3}{\text{kmol} \cdot \text{K}}$$

$$\frac{\bar{R}}{M} [=] 0.287 \frac{\text{KPa} \cdot \text{m}^3}{\text{kg} \cdot \text{K}}$$

$$V_1 = A * h \rightarrow 0.25 \text{ m}$$

$$\downarrow$$

$$\pi R^2 \rightarrow 0.05 \text{ m}$$

$$= 0.001963 \text{ m}^3$$

$$P V = n \bar{R} T$$

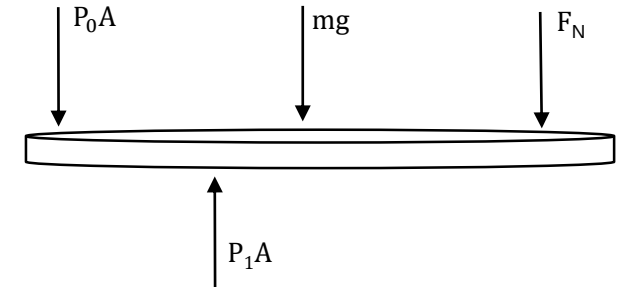
$$P V = \frac{m}{M} \bar{R} T, \quad \frac{\bar{R}}{M} = R$$

$$= R_g$$

A force balance (**STATE 1**) on the piston shows that (F_N is the force on the stops):

$$\begin{aligned} P \cdot A &\Rightarrow \text{Pa} \cdot \text{m}^2 = 1 \text{ N} \\ mg &\Rightarrow \frac{\text{kg} \cdot \text{m}}{\text{s}^2} = 1 \text{ N} \end{aligned}$$

$$P_1 A = P_0 A + mg + F_N$$



The piston just moves down (**STATE 2**) when F_N is zero and : $P_2 A = P_0 A + mg + 0$

$$P_2 = P_0 + \frac{mg}{A}$$

$$P_2 = 100000 + \frac{50 \times 9.81}{\pi \times 0.05^2} = 162452.2536 \text{ Pa} = 162.4523 \text{ kPa}$$

a) The temperature at that moment is:

$$T_2 = \frac{P_2 V}{mR} = \frac{162.4523 \times (\pi \times 0.05^2 \times 0.25)}{2.984 \times 10^{-3} \times \frac{8.314}{28.97}} = 372.4744 \text{ K} = 99.3244 \text{ } ^\circ\text{C}$$

$V_1 = V_2 \rightarrow$ Just start to move

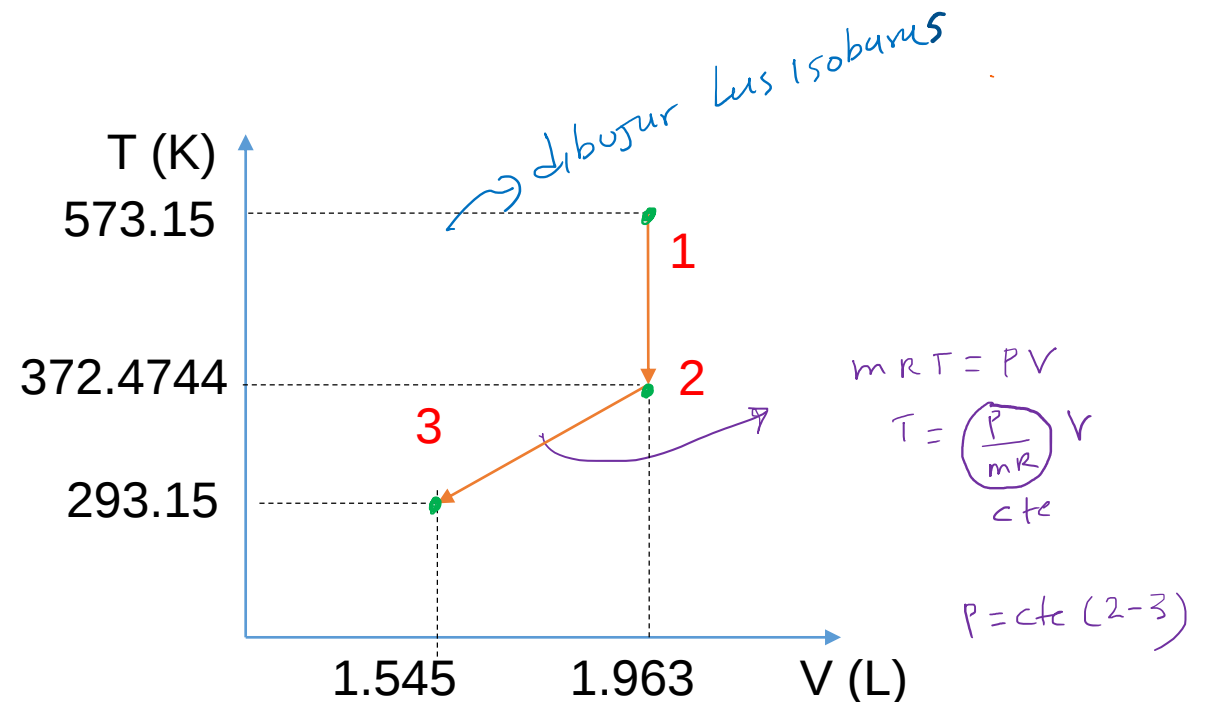
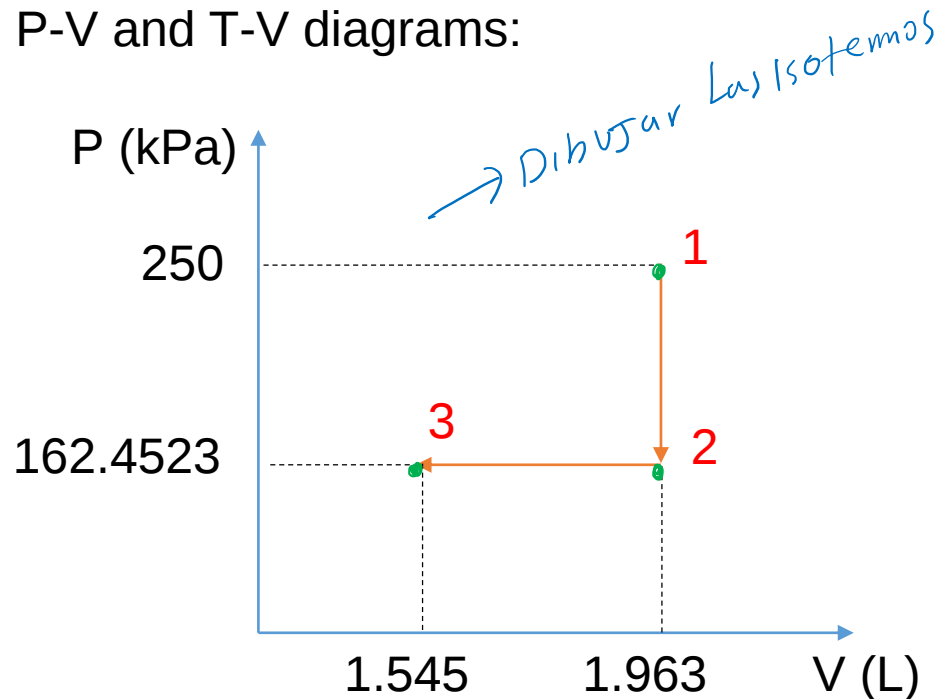
b) When the temperature is equal to ambient (20°C) (**STATE 3**), the volume is: [Remember **P** remain constant... $P_2 = P_3$]

$$V_3 = \frac{mRT_3}{P_3} = \frac{2.984 \times 10^{-3} \times \frac{8.314}{28.97} \times (20 + 273.15)}{162.4523} = 1.545 \times 10^{-3} \text{ m}^3$$

The height of the piston is:

$$h_3 = \frac{V_3}{A} = \frac{1.545 \times 10^{-3}}{\pi \times 0.05^2} = 0.1967 \text{ m} = 19.67 \text{ cm}$$

c) P-V and T-V diagrams:



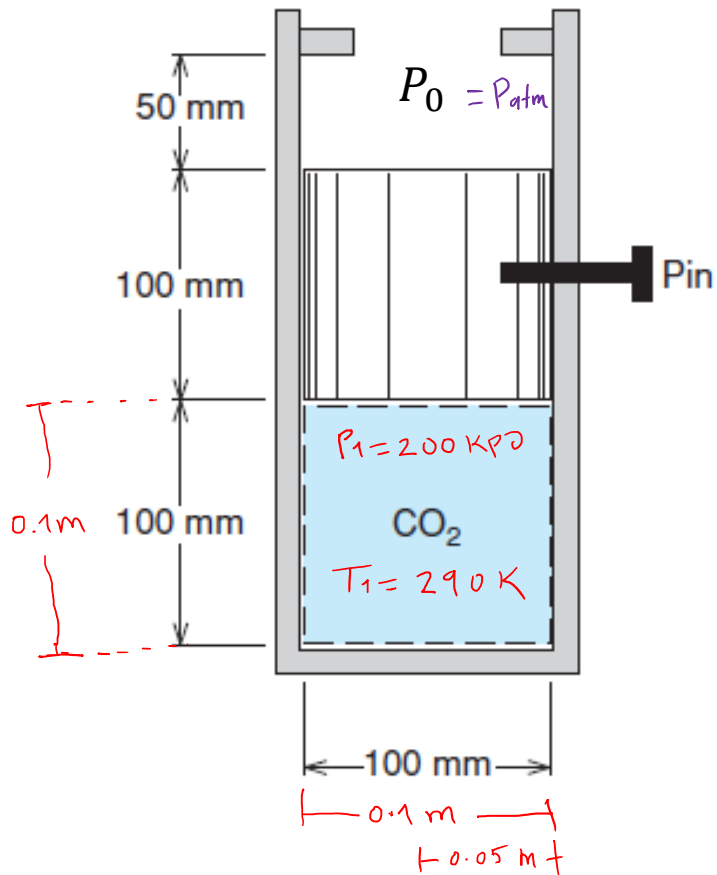
EXAMPLE 3.3

A cylinder has a thick piston initially held by a pin, as shown below. The cylinder contains carbon dioxide at 200 kPa and ambient temperature of 290 K. The metal piston has a density of 8000 kg/m^3 and the atmospheric pressure is 101 kPa . The pin is now removed, allowing the piston to move, and after a while the gas returns to ambient temperature. Is the piston against the stops?

$$\hookrightarrow T_1 = T_2$$

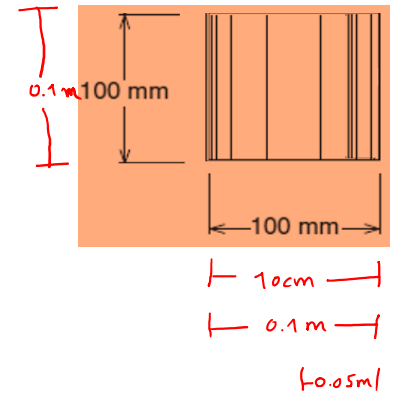
The first thing is to see if there is expansion once the pin is removed. We need to calculate the external pressure:

$$P_{ext} = P_0 + \frac{mg}{A} = P_0 + \frac{\rho V_{piston} g}{A}$$



$$P_{ext} = P_0 + \frac{mg}{A} = P_0 + \frac{\rho V_{piston} g}{A}$$

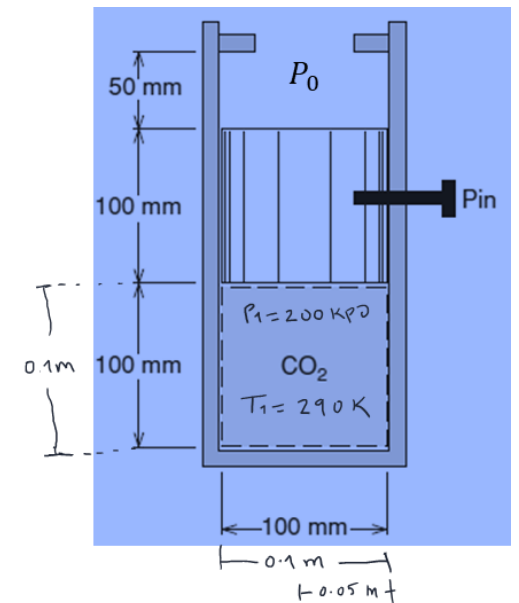
$$P_{ext} = 101000 + \frac{8000 \times \pi * 0.05^2 * 0.1 \times 9.81}{\pi * 0.052} = 108848 \text{ Pa} = 108.848 \text{ kPa}$$



In the **state 1** we can calculate the CO₂ mass:

$$m_{CO_2} = \frac{PV}{RT} = \frac{200 \times (\pi * 0.052 * 0.1)}{\frac{8.314}{44.01} \times 290} = 2.867 \times 10^{-3} \text{ kg}$$

$V_1 = 0.000785 \text{ m}^3$

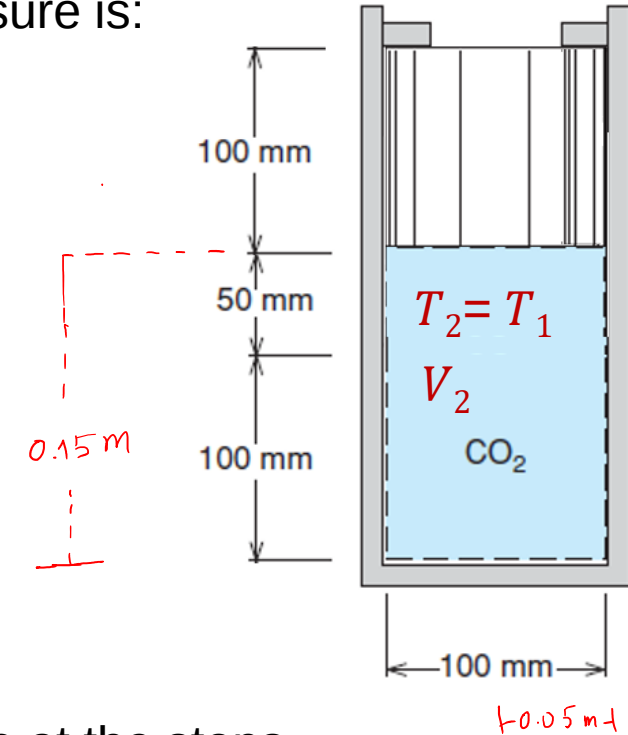


To check if the final state is at the stops or not, we have to estimate the pressure when the CO₂ occupies all the available volume and verify that the pressure at that point is larger than $P_{ext} = 108.848$ kPa:

Assuming the piston is at the stops ("state 2"), the pressure is:

$$P_2 = \frac{mRT_2}{V_2} = \frac{2.867 \times 10^{-3} \times \frac{8.314}{44.01} \times 290}{\pi * 0.052 * 0.15} = \mathbf{133.3221 \text{ kPa}}$$

$V_2 = 0.001178 \text{ m}^3$



As the pressure is larger than 108.848 kPa, the piston must be at the stops

Thermodynamic

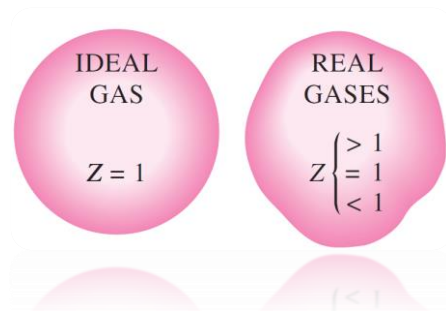
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S3-2_AULA

NEDHER SÁNCHEZ RAMÍREZ

NAUDY LOPEZ

Ideal and Real Gases



Session 5: Apply the ideal gas model for thermodynamic analysis using the equation of state and **estimating properties such as internal energy and enthalpy.**

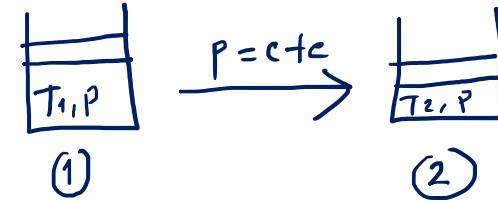
Session 6: Determine the properties (P-V-T) of a real gas with the application of the reduced temperature, the reduced pressure and the compressibility diagram.

u, h for Ideal Gas?

TABLE A-22

Ideal Gas Properties of Air

T(K), h and u(kJ/kg), s° (kJ/kg · K)										
T	when Δs = 0 ¹									
	h	u	s°	p _r	v _r	T	h	u	s°	when Δs = 0
200	199.97	142.56	1.29559	0.3363	1707.	450	451.80	322.62	2.11161	5.775
210	209.97	149.69	1.34444	0.3987	1512.	460	462.02	329.97	2.13407	6.245
220	219.97	156.82	1.39105	0.4690	1346.	470	472.24	337.32	2.15604	6.742
230	230.02	164.00	1.43557	0.5477	1205.	480	482.49	344.70	2.17760	7.268
240	240.02	171.13	1.47824	0.6355	1084.	490	492.74	352.08	2.19876	7.824



$$\Delta u \left[\frac{\text{kJ}}{\text{kg}} \right] = u_2 - u_1 \quad ?$$

$\downarrow$ T_2 $\downarrow$ T_1
 $\Delta h \left[\frac{\text{kJ}}{\text{kg}} \right] = h_2 - h_1 \quad ?$

TABLE A-23

Ideal Gas Properties of Selected Gases

Enthalpy $\bar{h}(T)$ and internal energy $\bar{u}(T)$, in kJ/kmol. Absolute entropy at 1 atm $\bar{s}^\circ(T)$, in kJ/kmol · K.																
T(K)	Carbon Dioxide, CO ₂ ($\bar{h}_f^\circ = -393,520$ kJ/kmol)			Carbon Monoxide, CO ($\bar{h}_f^\circ = -110,530$ kJ/kmol)			Water Vapor, H ₂ O ($\bar{h}_f^\circ = -241,820$ kJ/kmol)			Oxygen, O ₂ ($\bar{h}_f^\circ = 0$ kJ/kmol)			Nitrogen, N ₂ ($\bar{h}_f^\circ = 0$ kJ/kmol)			T(K)
	$\bar{h}$	$\bar{u}$	$\bar{s}^\circ$	$\bar{h}$	$\bar{u}$	$\bar{s}^\circ$	$\bar{h}$	$\bar{u}$	$\bar{s}^\circ$	$\bar{h}$	$\bar{u}$	$\bar{s}^\circ$	$\bar{h}$	$\bar{u}$	$\bar{s}^\circ$	
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
220	6,601	4,772	202.966	6,391	4,562	188.683	7,295	5,466	178.576	6,404	4,575	196.171	6,391	4,562	182.638	220
230	6,938	5,026	204.464	6,683	4,771	189.980	7,628	5,715	180.054	6,694	4,782	197.461	6,683	4,770	183.938	230
240	7,280	5,285	205.920	6,975	4,979	191.221	7,961	5,965	181.471	6,984	4,989	198.696	6,975	4,979	185.180	240
250	7,627	5,548	207.337	7,266	5,188	192.411	8,294	6,215	182.831	7,275	5,197	199.885	7,266	5,188	186.370	250
260	7,979	5,817	208.717	7,558	5,396	193.554	8,627	6,466	184.139	7,566	5,405	201.027	7,558	5,396	187.514	260
270	8,335	6,091	210.062	7,849	5,604	194.654	8,961	6,716	185.399	7,858	5,613	202.128	7,849	5,604	188.614	270

Is there another way?

$$du = c_v(T) dT$$

$$dh = c_p(T) dT$$

$\frac{\text{kJ}}{\text{kg}}$

INTERNAL ENERGY, ENTHALPY, AND SPECIFIC HEATS OF IDEAL GASES

$u = u(T, v)$ y tome su diferencial total:

$$du = \left(\frac{\partial u}{\partial T} \right)_v dT + \left(\frac{\partial u}{\partial v} \right)_T dv, \quad c_v = \left(\frac{\partial u}{\partial T} \right)_v$$

$$du = c_v dT + \left(\frac{\partial u}{\partial v} \right)_T dv$$

$$\begin{cases} h = u + Pv \\ Pv = R_g T \end{cases}$$



$$h = u + R_g T \Rightarrow h = f(T) \Rightarrow h = f(T, P), \quad dh = c_p dT$$

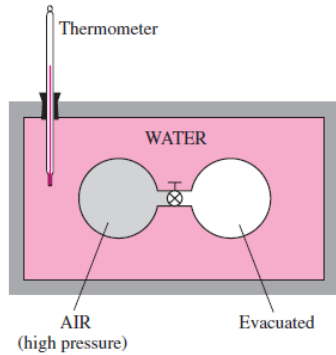


FIGURE 4-22

Schematic of the experimental apparatus used by Joule.

$m = 1 \text{ kg}$
 $\Delta T = 1^\circ\text{C}$
 Specific heat = $5 \text{ kJ/kg} \cdot ^\circ\text{C}$
 5 kJ

Specific heat is the energy required to raise the temperature of a unit mass of a substance by one degree in a specified way.

P
 V
 by T. Wayne

Isobaric



C_p



P
 V
 by T. Wayne

Isochoric



C_v



INTERNAL ENERGY, ENTHALPY, AND SPECIFIC HEATS OF IDEAL GASES

For ideal gases:

$$Pv = R_g T$$

$$h = u + Pv = u + R_g T \rightarrow u = h - R_g T$$

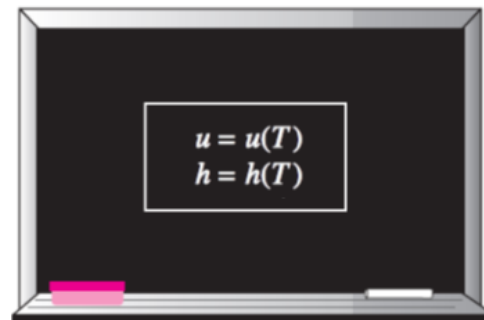
$$\therefore \Delta u = \Delta h - R_g \Delta T$$



$$c_p = c_v + R_g \quad (\text{kJ/kg} \cdot \text{K})$$

$$\bar{c}_p = \bar{c}_v + \bar{R} \quad (\text{kJ/kmol} \cdot \text{K})$$

demonstrated that for I.G. :



$$du = c_v(T) dT$$

$$dh = c_p(T) dT$$

4-5 ■ INTERNAL ENERGY, ENTHALPY, AND SPECIFIC HEATS OF SOLIDS AND LIQUIDS

A substance whose specific volume (or density) is constant is called an **incompressible substance**. The specific volumes of solids and liquids essentially remain constant during a process (Fig. 4-33). Therefore, liquids and solids can be approximated as incompressible substances without sacrificing much in accuracy. The constant-volume assumption should be taken to imply that the energy associated with the volume change is negligible compared with other forms of energy. Otherwise, this assumption would be ridiculous for studying the thermal stresses in solids (caused by volume change with temperature) or analyzing liquid-in-glass thermometers.

It can be mathematically shown that (see Chap. 12) the constant-volume and constant-pressure specific heats are identical for incompressible substances (Fig. 4-34). Therefore, for solids and liquids, the subscripts on c_p

and c_v can be dropped, and both specific heats can be represented by a single symbol c . That is,

$$c_p = c_v = c \quad (4-32)$$

This result could also be deduced from the physical definitions of constant-volume and constant-pressure specific heats. Specific heat values for several common liquids and solids are given in Table A-3.

Internal Energy Changes

Like those of ideal gases, the specific heats of incompressible substances depend on temperature only. Thus, the partial differentials in the defining equation of c_v can be replaced by ordinary differentials, which yield

$$du = c_v dT = c(T) dT \quad (4-33)$$

The change in internal energy between states 1 and 2 is then obtained by integration:

$$\Delta u = u_2 - u_1 = \int_1^2 c(T) dT \quad (\text{kJ/kg}) \quad (4-34)$$

The variation of specific heat c with temperature should be known before this integration can be carried out. For small temperature intervals, a c value at the average temperature can be used and treated as a constant, yielding

$$\Delta u \cong c_{\text{avg}}(T_2 - T_1) \quad (\text{kJ/kg}) \quad (4-35)$$

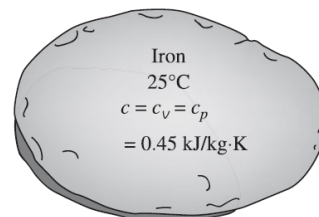


FIGURE 4-34

The c_v and c_p values of incompressible substances are identical and are denoted by c .

Enthalpy Changes

Using the definition of enthalpy $h = u + Pv$ and noting that $v = \text{constant}$, the differential form of the enthalpy change of incompressible substances can be determined by differentiation to be

$$dh = du + v dP + P d\underbrace{v}_0 = du + v dP \quad (4-36)$$

Integrating,

$$\Delta h = \Delta u + v \Delta P \cong c_{\text{avg}} \Delta T + v \Delta P \quad (\text{kJ/kg}) \quad (4-37)$$

For *solids*, the term $v \Delta P$ is insignificant and thus $\Delta h = \Delta u \cong c_{\text{avg}} \Delta T$. For *liquids*, two special cases are commonly encountered:

1. *Constant-pressure processes*, as in heaters ($\Delta P = 0$): $\Delta h = \Delta u \cong c_{\text{avg}} \Delta T$
2. *Constant-temperature processes*, as in pumps ($\Delta T = 0$): $\Delta h = v \Delta P$

For a process between states 1 and 2, the last relation can be expressed as $h_2 - h_1 = v(P_2 - P_1)$. By taking state 2 to be the compressed liquid state at a given T and P and state 1 to be the saturated liquid state at the same temperature, the enthalpy of the compressed liquid can be expressed as

$$h_{@P,T} \cong h_{f@T} + v_{f@T}(P - P_{\text{sat}@T}) \quad (4-38)$$

as discussed in Chap. 3. This is an improvement over the assumption that the enthalpy of the compressed liquid could be taken as h_f at the given temperature (that is, $h_{@P,T} \cong h_{f@T}$). However, the contribution of the last term

is often very small, and is neglected. (Note that at high temperature and pressures, Eq. 4-38 may overcorrect the enthalpy and result in a larger error than the approximation $h \cong h_{f@T}$.)

2. The difference between c_p and c_v approaches zero as the absolute temperature approaches zero.
3. The two specific heats are identical for truly incompressible substances since $v = \text{constant}$. The difference between the two specific heats is very small and is usually disregarded for substances that are *nearly* incompressible, such as liquids and solids.

INTERNAL ENERGY, ENTHALPY, AND SPECIFIC HEATS OF IDEAL GASES

$$du = c_v(T) dT$$

$$dh = c_p(T) dT$$

Note that the du and dh relations given previously are not restricted to any kind of process. They are valid for all processes.

HOW TO EVALUATE INTERNAL ENERGY AND ENTHALPY OF IDEAL GASES?

Equivalentes en términos de precisión

1. Tabulated data and calculating Δh : Tables A-22 y Table A-23

$$\Delta h = h_2(T_2) - h_1(T_1)$$

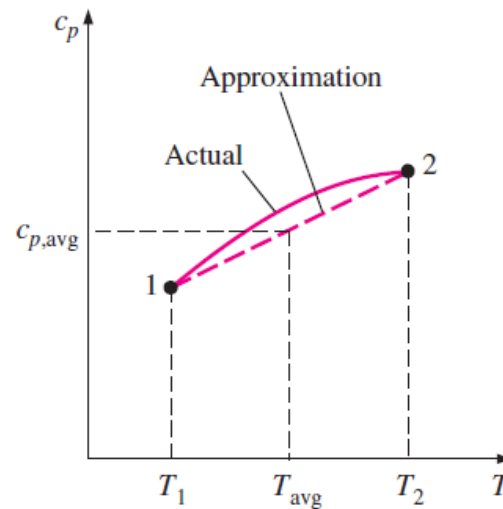
2. Using expression for $c_p(T)$ and integrating: Table A-21

$$\Delta h = h_2 - h_1 = \int_1^2 c_p(T) dT \quad (\text{kJ/kg}) = \int_1^2 \underbrace{(2T + 3T^2)}_{c_p(T)} dT$$

3. Using $c_{p,avg}$ and integrating Table A-20

$$\Delta h = C_{p,avg} \Delta T$$

Assume C_p cte con la T



OJO: Intervalos pequeños de T
de lo contrario toca usar el
método 2.

$\Delta h =$

HOW TO EVALUATE INTERNAL ENERGY AND ENTHALPY OF IDEAL GASES?

Equivalentes en términos de precisión

$\Delta h =$

1. Tabulated data and calculating Δh : Tables A-22 y Table A-23

$$\Delta h = h_2(T_2) - h_1(T_1)$$

2. Using expression for $c_p(T)$ and integrating: Table A-21

$$\Delta h = h_2 - h_1 = \int_1^2 c_p(T) dT \quad (\text{kJ/kg}) = \int (2T + 3T^2) dT$$

$\underbrace{\hspace{10em}}_{c_p(T)}$

3. Using c_{p_avg} and integrating Table A-20

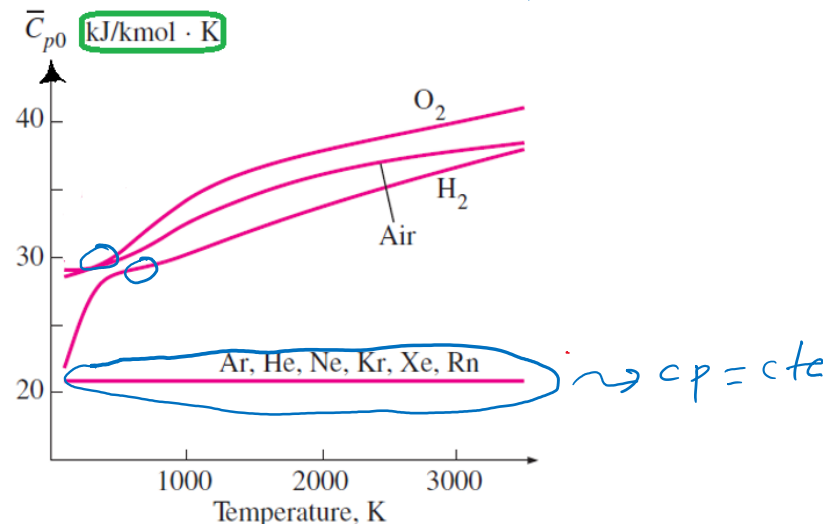
$$\Delta h = C_{p_avg} \Delta T$$

$\hookrightarrow$ Assume C_p cte con la T

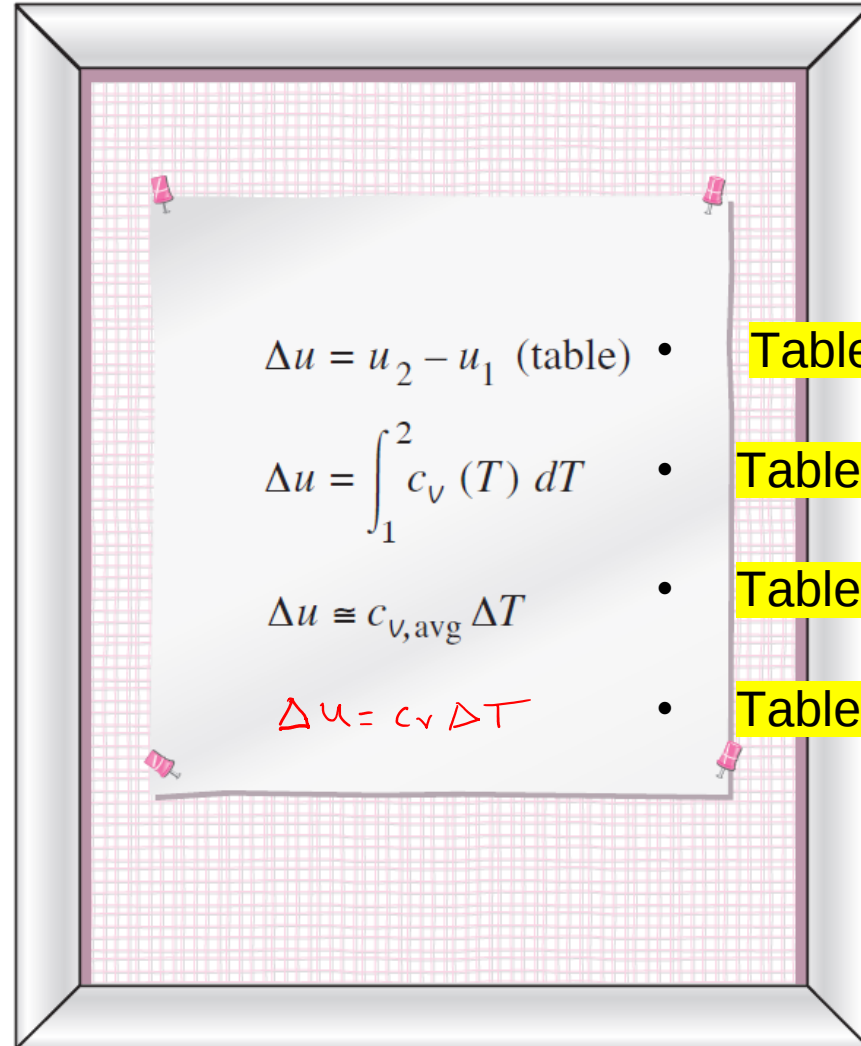
4. Using c_p constant and integrating: Table A-20

$$\Delta h = c_p \Delta T \rightarrow h = c_p T$$

$\hookrightarrow$ Assume C_p cte con la T



In the same way.....



Tables A-22 y Table A-23

Table A-21

→ C_p variable → ver ejemplo 9-1

Table A-20

→ c_v cte

Table A-20

→ c_v cte

8.1. Determine the enthalpy change h of nitrogen, in **kJ/kg**, as it is heated from **400** to **800** K, using (a) the empirical specific heat equation as a function of temperature (Table A–21), (b) the c_p value at the average temperature (Table A–20), and (c) the c_p value at 400K (Table A–20). (d) Using the table A-23.

a) c_p as function of temperature – Table A-21

TABLE A-21

Variation of $\bar{c}_p$ with Temperature for Selected Ideal Gases

$$\frac{\bar{c}_p}{\bar{R}} = \alpha + \beta T + \gamma T^2 + \delta T^3 + \varepsilon T^4 \rightarrow \text{Dimensional}$$

T is in K, equations valid from 300 to 1000 K

Gas	α	$\beta \times 10^3$	$\gamma \times 10^6$	$\delta \times 10^9$	$\varepsilon \times 10^{12}$
CO	3.710	–1.619	3.692	–2.032	0.240
CO ₂	2.401	8.735	–6.607	2.002	0
H ₂	3.057	2.677	–5.810	5.521	–1.812
H ₂ O	4.070	–1.108	4.152	–2.964	0.807
O ₂	3.626	–1.878	7.055	–6.764	2.156
N ₂	3.675	–1.208	2.324	–0.632	–0.226
Air	3.653	–1.337	3.294	–1.913	0.2763
SO ₂	3.267	5.324	0.684	–5.281	2.559
CH ₄	3.826	–3.979	24.558	–22.733	6.963
C ₂ H ₂	1.410	19.057	–24.501	16.391	–4.135
C ₂ H ₄	1.426	11.383	7.989	–16.254	6.749
Monatomic gases ^a	2.5	0	0	0	0

$$\Delta \bar{h} = \int \bar{c}_p dT$$

$\frac{\text{kJ}}{\text{kmol K}}$
 $\bar{R} = 8.314 \frac{\text{kJ}}{\text{kmol K}}$

$$\Delta \bar{h} = \int_{400}^{800} \bar{R}(\alpha + \beta T + \gamma T^2 + \delta T^3 + \varepsilon T^4) dT$$

$$\Delta \bar{h} = \bar{R} \left[\alpha T + \beta \frac{T^2}{2} + \gamma \frac{T^3}{3} + \delta \frac{T^4}{4} + \varepsilon \frac{T^5}{5} \right]_{400}^{800}$$

$$\Delta \bar{h} = \bar{R} \left[3.675(800 - 400) - 1.208 \times 10^{-3} \left(\frac{800^2 - 400^2}{2} \right) + 2.324 \times 10^{-6} \left(\frac{800^3 - 400^3}{3} \right) - 0.632 \times 10^{-9} \left(\frac{800^4 - 400^4}{4} \right) - 0.226 \times 10^{-12} \left(\frac{800^5 - 400^5}{5} \right) \right]$$

$$\Delta \bar{h} = \bar{R}[1452.1104][=] \frac{\text{kJ}}{\text{kmol}}$$

$$\Delta h = \frac{8.314}{28.01} \times 1452.1104 \frac{\text{kJ}}{\text{kg}} = 431.0191 \frac{\text{kJ}}{\text{kg}}$$

b) c_p at an average temperature $= (800+400)/2 = 600\text{K}$

TABLE A-20

Ideal Gas Specific Heats of Some Common Gases (kJ/kg · K)

Temp. K	c_p	c_v	k	c_p	c_v	k	c_p	c_v	k	Temp. K
	Air			Nitrogen, N ₂			Oxygen, O ₂			
250	1.003	0.716	1.401	1.039	0.742	1.400	0.913	0.653	1.398	250
300	1.005	0.718	1.400	1.039	0.743	1.400	0.918	0.658	1.395	300
350	1.008	0.721	1.398	1.041	0.744	1.399	0.928	0.668	1.389	350
400	1.013	0.726	1.395	1.044	0.747	1.397	0.941	0.681	1.382	400
450	1.020	0.733	1.391	1.049	0.752	1.395	0.956	0.696	1.373	450
500	1.029	0.742	1.387	1.056	0.759	1.391	0.972	0.712	1.365	500
550	1.040	0.753	1.381	1.065	0.768	1.387	0.988	0.728	1.358	550
600	1.051	0.764	1.376	1.075	0.778	1.382	1.003	0.743	1.350	600
650	1.063	0.776	1.370	1.086	0.789	1.376	1.017	0.758	1.343	650
700	1.075	0.788	1.364	1.098	0.801	1.371	1.031	0.771	1.337	700
750	1.087	0.800	1.359	1.110	0.813	1.365	1.043	0.783	1.332	750
800	1.099	0.812	1.354	1.121	0.825	1.360	1.054	0.794	1.327	800
900	1.121	0.834	1.344	1.145	0.849	1.349	1.074	0.814	1.319	900
1000	1.142	0.855	1.336	1.167	0.870	1.341	1.090	0.830	1.313	1000

$$\Delta h = \int c_p dT = c_{p,avg}(\Delta T)$$

$$\Delta h = 1.075 \times (800 - 400) = 430 \frac{\text{kJ}}{\text{kg}}$$

c) c_p at 400K

TABLE A-20										
Ideal Gas Specific Heats of Some Common Gases (kJ/kg · K)										
Temp. K	c_p	c_v	k	c_p	c_v	k	c_p	c_v	k	Temp. K
	Air			Nitrogen, N ₂			Oxygen, O ₂			
250	1.003	0.716	1.401	1.039	0.742	1.400	0.913	0.653	1.398	250
300	1.005	0.718	1.400	1.039	0.743	1.400	0.918	0.658	1.395	300
350	1.008	0.721	1.398	1.041	0.744	1.399	0.928	0.668	1.389	350
400	1.013	0.726	1.395	1.044	0.747	1.397	0.941	0.681	1.382	400
450	1.020	0.733	1.391	1.049	0.752	1.395	0.956	0.696	1.373	450
500	1.029	0.742	1.387	1.056	0.759	1.391	0.972	0.712	1.365	500
550	1.040	0.753	1.381	1.065	0.768	1.387	0.988	0.728	1.358	550
600	1.051	0.764	1.376	1.075	0.778	1.382	1.003	0.743	1.350	600
650	1.063	0.776	1.370	1.086	0.789	1.376	1.017	0.758	1.343	650
700	1.075	0.788	1.364	1.098	0.801	1.371	1.031	0.771	1.337	700
750	1.087	0.800	1.359	1.110	0.813	1.365	1.043	0.783	1.332	750
800	1.099	0.812	1.354	1.121	0.825	1.360	1.054	0.794	1.327	800
900	1.121	0.834	1.344	1.145	0.849	1.349	1.074	0.814	1.319	900
1000	1.142	0.855	1.336	1.167	0.870	1.341	1.090	0.830	1.313	1000

$$\Delta h = \int c_p dT = c_p(\Delta T)$$

$$\Delta h = 1.044 \times (800 - 400) = 417.6 \frac{\text{kJ}}{\text{kg}}$$

d) Ideal gas enthalpy

TABLE A-23																
Ideal Gas Properties of Selected Gases																
Enthalpy $\bar{h}(T)$ and internal energy $\bar{u}(T)$, in kJ/kmol. Absolute entropy at 1 atm $\bar{s}^\circ(T)$, in kJ/kmol · K.																
$T(K)$	Carbon Dioxide, CO ₂ ($\bar{h}_f^\circ = -393,520$ kJ/kmol)			Carbon Monoxide, CO ($\bar{h}_f^\circ = -110,530$ kJ/kmol)			Water Vapor, H ₂ O ($\bar{h}_f^\circ = -241,820$ kJ/kmol)			Oxygen, O ₂ ($\bar{h}_f^\circ = 0$ kJ/kmol)			Nitrogen, N ₂ ($\bar{h}_f^\circ = 0$ kJ/kmol)			$T(K)$
	$\bar{h}$	$\bar{u}$	$\bar{s}^\circ$	$\bar{h}$	$\bar{u}$	$\bar{s}^\circ$	$\bar{h}$	$\bar{u}$	$\bar{s}^\circ$	$\bar{h}$	$\bar{u}$	$\bar{s}^\circ$	$\bar{h}$	$\bar{u}$	$\bar{s}^\circ$	
400	13,372	10,046	225.225	11,644	8,319	206.125	13,356	10,030	198.673	11,711	8,384	213.765	11,640	8,314	200.071	400
800	32,179	25,527	257.408	23,844	17,193	227.162	27,896	21,245	223.693	24,523	17,872	235.810	23,714	17,061	220.907	800

$$\Delta \bar{h} = \bar{h}_2 - \bar{h}_1$$

$$\Delta \bar{h} = 23714 - 11640 = 12074 \frac{\text{kJ}}{\text{kmol}}$$

$$\Delta h = 12074 \frac{\text{kJ}}{\text{kmol}} \times \frac{1}{28.01} \frac{\text{kmol}}{\text{kg}}$$

$$\Delta h = 431.0603 \frac{\text{kJ}}{\text{kg}}$$

9.1. Determine the internal energy change u of carbon dioxide, in kJ/kg, as it is heated from 300 to 900 K, using **(a)** the empirical specific heat equation as a function of temperature (Table A–21), **(b)** the c_v value at the average temperature (Table A–20), and **(c)** the c_v value at room temperature (300K) (Table A–20). **(d)** Using the table A-23.

a) c_p as function of temperature – Table A-21

TABLE A-21					
Variation of $\bar{c}_p$ with Temperature for Selected Ideal Gases					
$\frac{\bar{c}_p}{R} = \alpha + \beta T + \gamma T^2 + \delta T^3 + \epsilon T^4$					
T is in K, equations valid from 300 to 1000 K					
Gas	α	$\beta \times 10^3$	$\gamma \times 10^6$	$\delta \times 10^9$	$\epsilon \times 10^{12}$
CO	3.710	−1.619	3.692	−2.032	0.240
CO ₂	2.401	8.735	−6.607	2.002	0
H ₂	3.057	2.677	−5.810	5.521	−1.812
H ₂ O	4.070	−1.108	4.152	−2.964	0.807
O ₂	3.626	−1.878	7.055	−6.764	2.156
N ₂	3.675	−1.208	2.324	−0.632	−0.226
Air	3.653	−1.337	3.294	−1.913	0.2763
SO ₂	3.267	5.324	0.684	−5.281	2.559
CH ₄	3.826	−3.979	24.558	−22.733	6.963
C ₂ H ₂	1.410	19.057	−24.501	16.391	−4.135
C ₂ H ₄	1.426	11.383	7.989	−16.254	6.749
Monatomic gases ^a	2.5	0	0	0	0

For ideal gases:

$$h = u + Pv = u + RT \rightarrow u = h - RT$$

$$\therefore \Delta u = \Delta h - R\Delta T$$

$$\Delta \bar{h} = \int \bar{c}_p dT$$

$$\Delta \bar{h} = \int_{300}^{900} R(\alpha + \beta T + \gamma T^2 + \delta T^3 + \epsilon T^4) dT$$

$$\Delta \bar{h} = R \left[\alpha T + \beta \frac{T^2}{2} + \gamma \frac{T^3}{3} + \delta \frac{T^4}{4} + \epsilon \frac{T^5}{5} \right]_{300}^{900}$$

$$\Delta \bar{h} = \bar{R} \left[2.401(900 - 300) + 8.735 \times 10^{-3} \left(\frac{900^2 - 300^2}{2} \right) - 6.607 \times 10^{-6} \left(\frac{900^3 - 300^3}{3} \right) + 2.002 \times 10^{-9} \left(\frac{900^4 - 300^4}{4} \right) + 0.000 \times 10^{-12} \left(\frac{900^5 - 300^5}{5} \right) \right]$$

$$\Delta \bar{h} = R[3363.486] \dots \Delta h = \frac{8.314}{44.01} \times 3363.486 \frac{\text{kJ}}{\text{kg}} = 635.4016 \frac{\text{kJ}}{\text{kg}}$$

$$\therefore \Delta u = \Delta h - R\Delta T = 635.40 - \frac{8.314}{44.01} \times (900 - 300) = 522.0546 \frac{\text{kJ}}{\text{kg}}$$

b) c_v at an average temperature = 600K

TABLE A-20

Ideal Gas Specific Heats of Some Common Gases (kJ/kg · K)

	c_p	c_v	k	c_p	c_v	k	c_p	c_v	k	
Temp. K	Carbon Dioxide, CO ₂			Carbon Monoxide, CO			Hydrogen, H ₂			Temp. K
250	0.791	0.602	1.314	1.039	0.743	1.400	14.051	9.927	1.416	250
300	0.846	0.657	1.288	1.040	0.744	1.399	14.307	10.183	1.405	300
350	0.895	0.706	1.268	1.043	0.746	1.398	14.427	10.302	1.400	350
400	0.939	0.750	1.252	1.047	0.751	1.395	14.476	10.352	1.398	400
450	0.978	0.790	1.239	1.054	0.757	1.392	14.501	10.377	1.398	450
500	1.014	0.825	1.229	1.063	0.767	1.387	14.513	10.389	1.397	500
550	1.046	0.857	1.220	1.075	0.778	1.382	14.530	10.405	1.396	550
600	1.075	0.886	1.213	1.087	0.790	1.376	14.546	10.422	1.396	600
650	1.102	0.913	1.207	1.100	0.803	1.370	14.571	10.447	1.395	650
700	1.126	0.937	1.202	1.113	0.816	1.364	14.604	10.480	1.394	700
750	1.148	0.959	1.197	1.126	0.829	1.358	14.645	10.521	1.392	750
800	1.169	0.980	1.193	1.139	0.842	1.353	14.695	10.570	1.390	800
900	1.204	1.015	1.186	1.163	0.866	1.343	14.822	10.698	1.385	900
1000	1.234	1.045	1.181	1.185	0.888	1.335	14.983	10.859	1.380	1000

$$\Delta u = \int c_v dT = c_{v,avg}(\Delta T)$$

$$\Delta u = 0.886 \times (600) = 531.6 \frac{\text{kJ}}{\text{kg}}$$

c) c_v at room temperature = 300K

TABLE A-20

Ideal Gas Specific Heat

	c_p	c_v
Temp. K	Carbon Dioxide, CO ₂	
250	0.791	0.602
300	0.846	0.657

$$\Delta u = \int c_v dT = c_{v,avg}(\Delta T)$$

$$\Delta u = 0.657 \times (600) = 394.2 \frac{\text{kJ}}{\text{kg}}$$

d) Ideal gas internal energy

TABLE A-23

Ideal Gas Properties of Selected Gases

Enthalpy $\bar{h}(T)$ and internal energy $\bar{u}(T)$, in kJ/kmol. Absolute entropy at 1 atm $\bar{s}^0(T)$, in kJ/kmol · K.

T(K)	Carbon Dioxide, CO ₂ ($\bar{h}_f^0 = -393,520$ kJ/kmol)			Carbon Monoxide, CO ($\bar{h}_f^0 = -110,530$ kJ/kmol)			Water Vapor, H ₂ O ($\bar{h}_f^0 = -241,820$ kJ/kmol)		
	$\bar{h}$	$\bar{u}$	$\bar{s}^0$	$\bar{h}$	$\bar{u}$	$\bar{s}^0$	$\bar{h}$	$\bar{u}$	$\bar{s}^0$
300	9,431	6,939	213.915	8,723	6,229	197.723	9,966	7,472	188.928
900	37,405	29,922	263.559	27,066	19,583	230.957	31,828	24,345	228.321

$$\Delta u = \int c_v dT = u_2 - u_1$$

$$\Delta u = 29922 - 6939 = 22983 \frac{\text{kJ}}{\text{kmol}}$$

$$\Delta u = 22983 \frac{\text{kJ}}{\text{kmol}} \times \frac{1}{44.01} \frac{\text{kmol}}{\text{kg}}$$

$$\Delta u = 522.2222 \frac{\text{kJ}}{\text{kg}}$$

4-5 ■ INTERNAL ENERGY, ENTHALPY, AND SPECIFIC HEATS OF SOLIDS AND LIQUIDS

A substance whose specific volume (or density) is constant is called an **incompressible substance**. The specific volumes of solids and liquids essentially remain constant during a process (Fig. 4-33). Therefore, liquids and solids can be approximated as incompressible substances without sacrificing much in accuracy. The constant-volume assumption should be taken to imply that the energy associated with the volume change is negligible compared with other forms of energy. Otherwise, this assumption would be ridiculous for studying the thermal stresses in solids (caused by volume change with temperature) or analyzing liquid-in-glass thermometers.

It can be mathematically shown that (see Chap. 12) the constant-volume and constant-pressure specific heats are identical for incompressible substances (Fig. 4-34). Therefore, for solids and liquids, the subscripts on c_p

and c_v can be dropped, and both specific heats can be represented by a single symbol c . That is,

$$c_p = c_v = c \quad (4-32)$$

This result could also be deduced from the physical definitions of constant-volume and constant-pressure specific heats. Specific heat values for several common liquids and solids are given in Table A-3.

Internal Energy Changes

Like those of ideal gases, the specific heats of incompressible substances depend on temperature only. Thus, the partial differentials in the defining equation of c_v can be replaced by ordinary differentials, which yield

$$du = c_v dT = c(T) dT \quad (4-33)$$

The change in internal energy between states 1 and 2 is then obtained by integration:

$$\Delta u = u_2 - u_1 = \int_1^2 c(T) dT \quad (\text{kJ/kg}) \quad (4-34)$$

The variation of specific heat c with temperature should be known before this integration can be carried out. For small temperature intervals, a c value at the average temperature can be used and treated as a constant, yielding

$$\Delta u \cong c_{\text{avg}}(T_2 - T_1) \quad (\text{kJ/kg}) \quad (4-35)$$

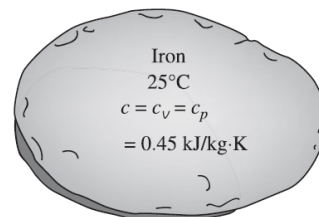


FIGURE 4-34

The c_v and c_p values of incompressible substances are identical and are denoted by c .

Enthalpy Changes

Using the definition of enthalpy $h = u + Pv$ and noting that $v = \text{constant}$, the differential form of the enthalpy change of incompressible substances can be determined by differentiation to be

$$dh = du + v dP + P d\overset{0}{v} = du + v dP \quad (4-36)$$

Integrating,

$$\Delta h = \Delta u + v \Delta P \cong c_{\text{avg}} \Delta T + v \Delta P \quad (\text{kJ/kg}) \quad (4-37)$$

For *solids*, the term $v \Delta P$ is insignificant and thus $\Delta h = \Delta u \cong c_{\text{avg}} \Delta T$. For *liquids*, two special cases are commonly encountered:

1. *Constant-pressure processes*, as in heaters ($\Delta P = 0$): $\Delta h = \Delta u \cong c_{\text{avg}} \Delta T$
2. *Constant-temperature processes*, as in pumps ($\Delta T = 0$): $\Delta h = v \Delta P$

For a process between states 1 and 2, the last relation can be expressed as $h_2 - h_1 = v(P_2 - P_1)$. By taking state 2 to be the compressed liquid state at a given T and P and state 1 to be the saturated liquid state at the same temperature, the enthalpy of the compressed liquid can be expressed as

$$h_{@P,T} \cong h_{f@T} + v_{f@T}(P - P_{\text{sat}@T}) \quad (4-38)$$

as discussed in Chap. 3. This is an improvement over the assumption that the enthalpy of the compressed liquid could be taken as h_f at the given temperature (that is, $h_{@P,T} \cong h_{f@T}$). However, the contribution of the last term

is often very small, and is neglected. (Note that at high temperature and pressures, Eq. 4-38 may overcorrect the enthalpy and result in a larger error than the approximation $h \cong h_{f@T}$.)

2. The difference between c_p and c_v approaches zero as the absolute temperature approaches zero.
3. The two specific heats are identical for truly incompressible substances since $v = \text{constant}$. The difference between the two specific heats is very small and is usually disregarded for substances that are *nearly* incompressible, such as liquids and solids.

At low pressures, all real gases approach ideal-gas behavior, and therefore their specific heats depend on temperature only.

