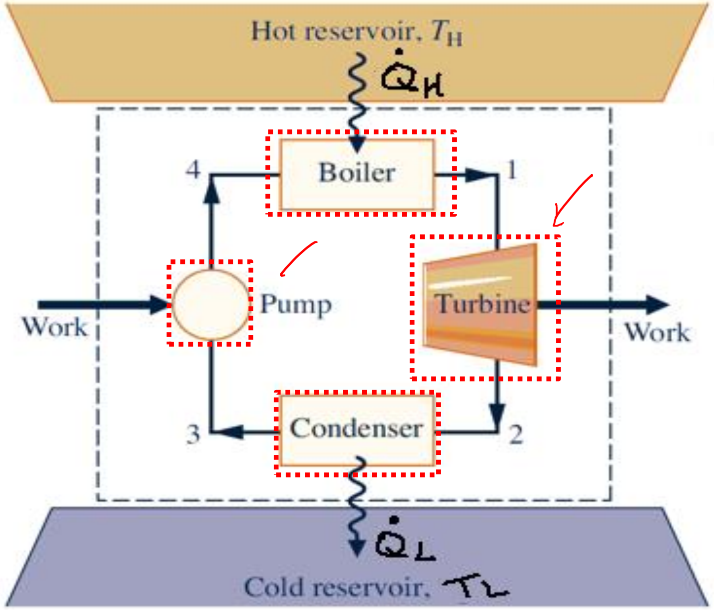
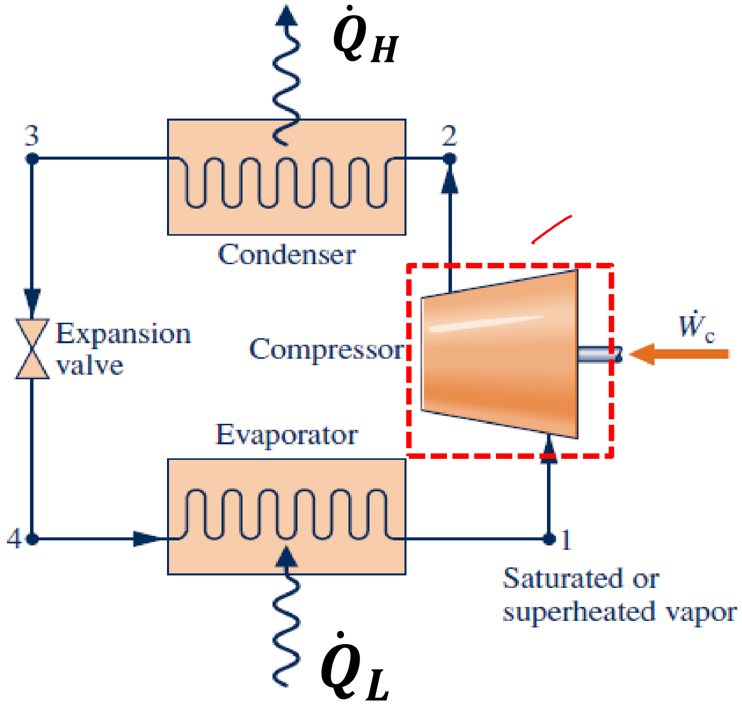
OBJECTIVE

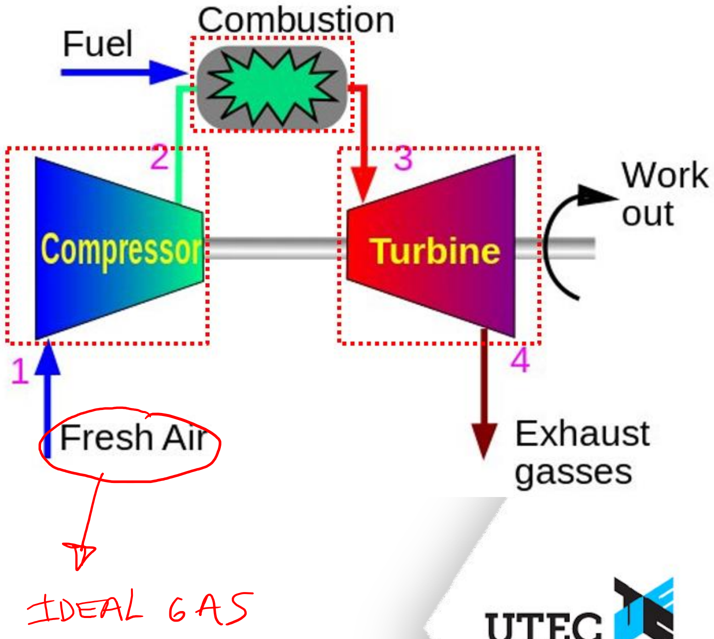
Session 27-28: Isentropic efficiency for turbines and compressors using ideal gases.

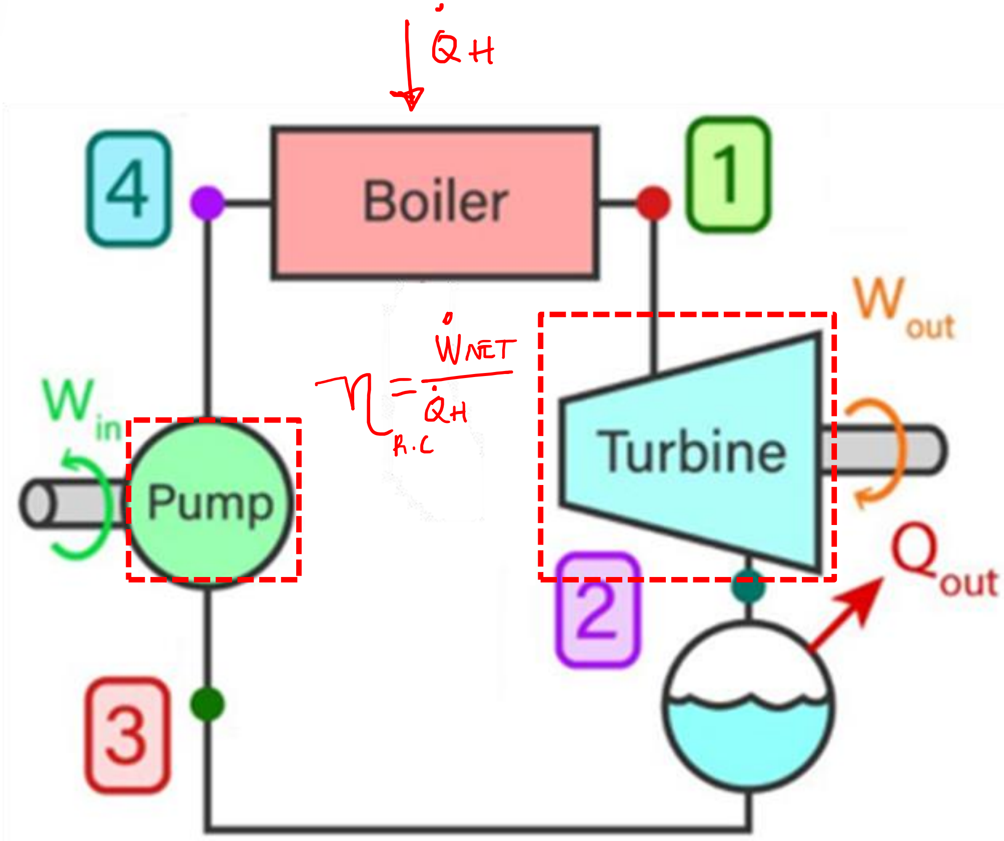
Why do we want to study the isentropic efficiency of turbines, compressors and pumps?



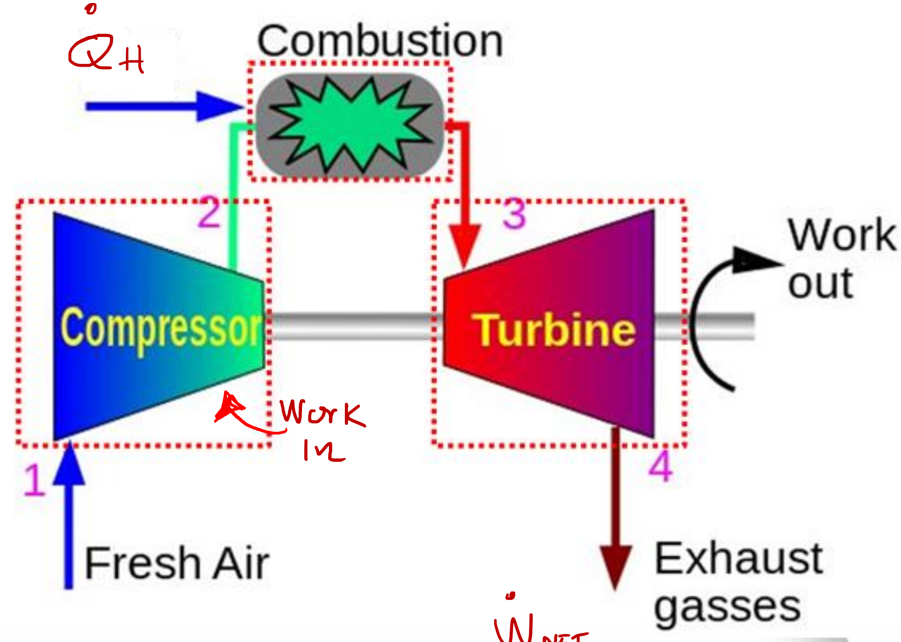


THIS WEEK



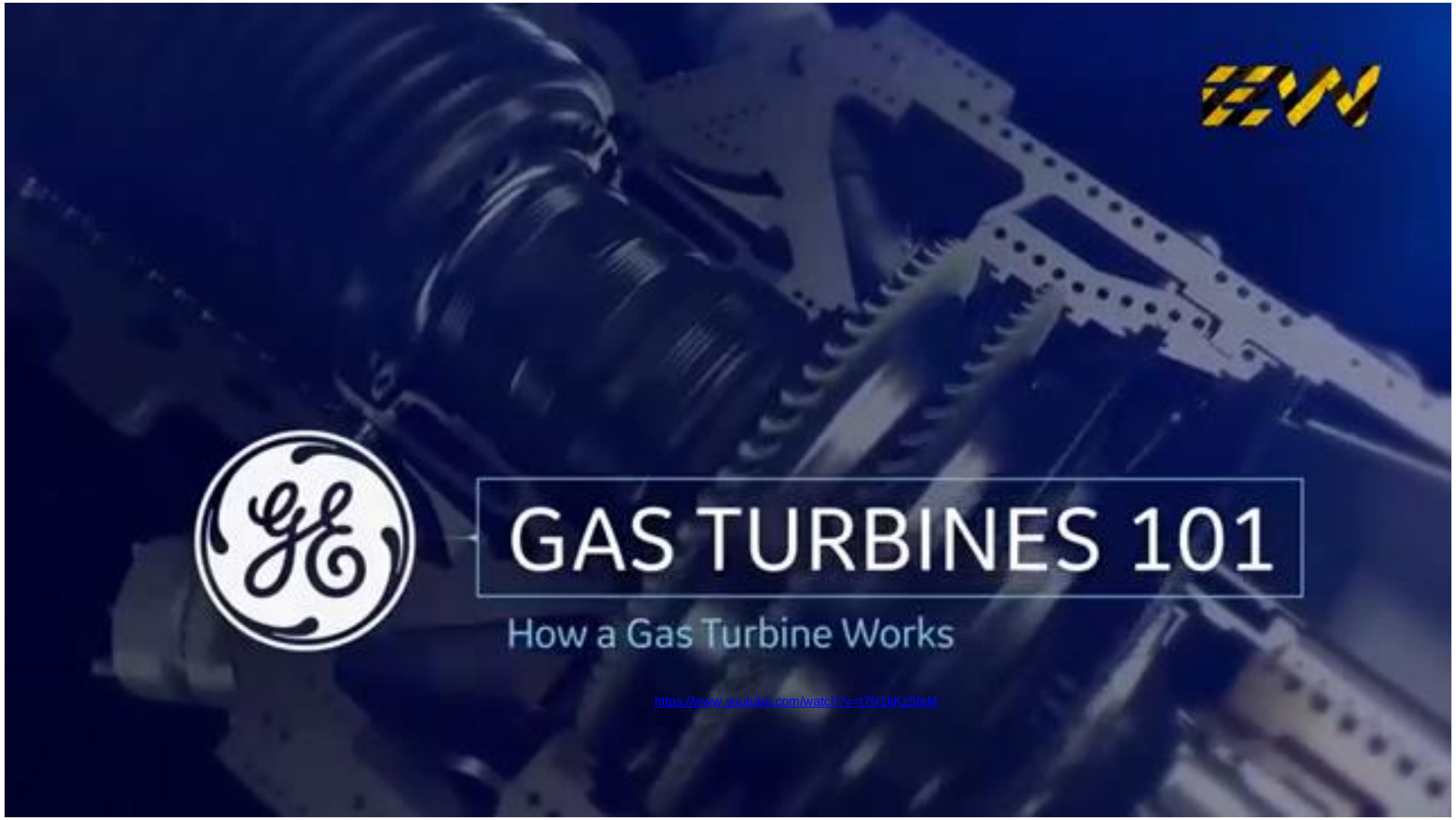


$$\eta_{R.C} = \frac{\dot{W}_{NET}}{\dot{Q}_H}$$



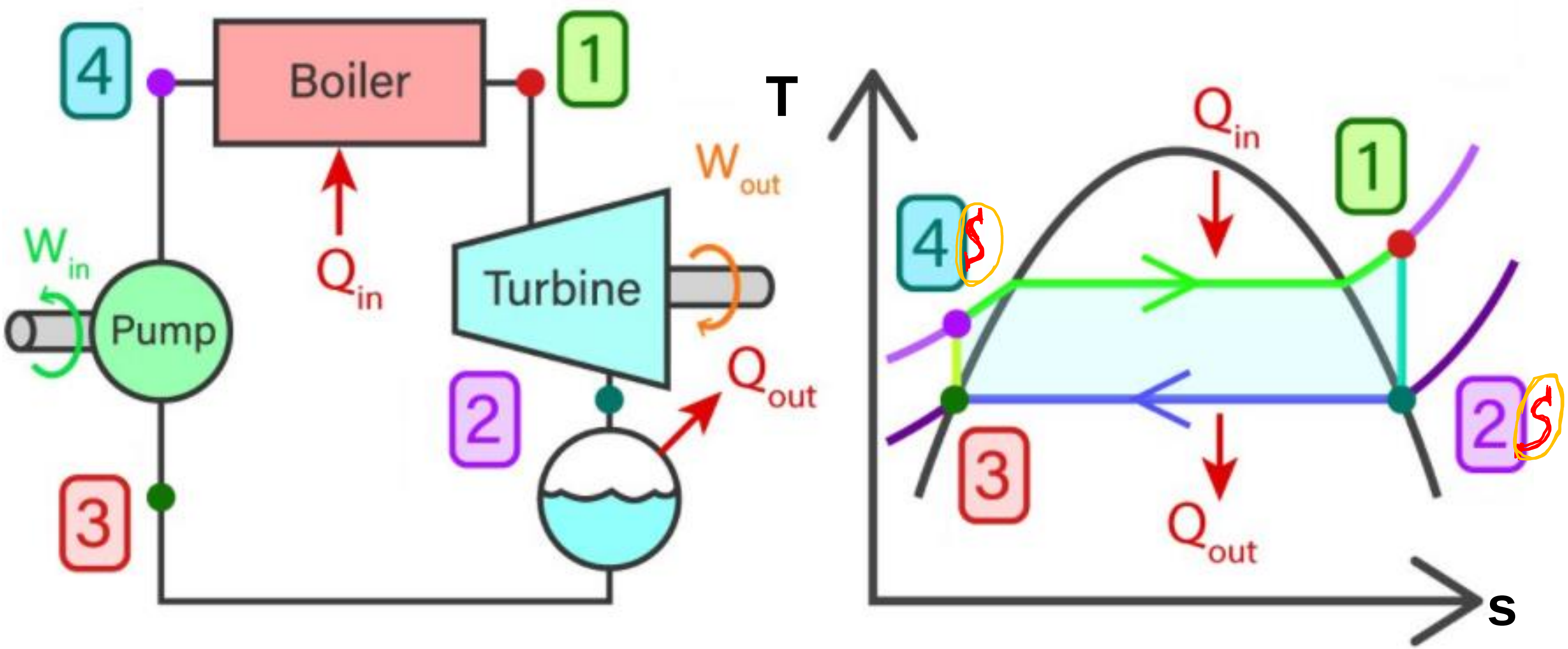
$$\eta_{BC} = \frac{\dot{W}_{NET}}{\dot{Q}_H}$$

Both are used to produce power, which one is better?



https://www.youtube.com/watch?v=t7N1kKz5faM

THE SIMPLE IDEAL RANKINE CYCLE_ Heat Engine with LV substance



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IDEAL BRAYTON CYCLE

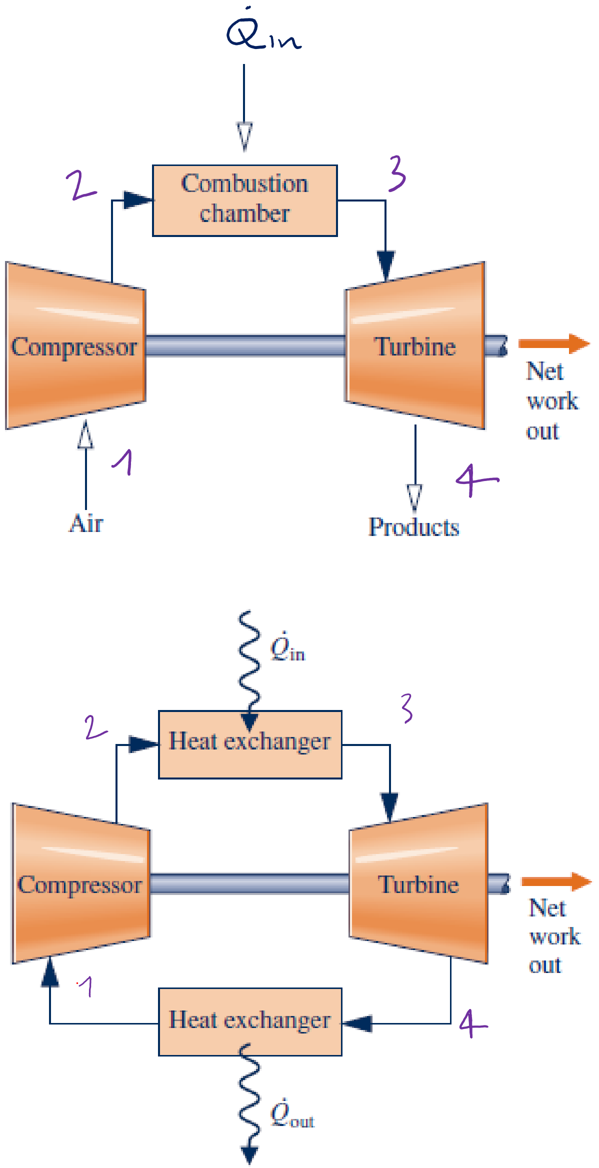
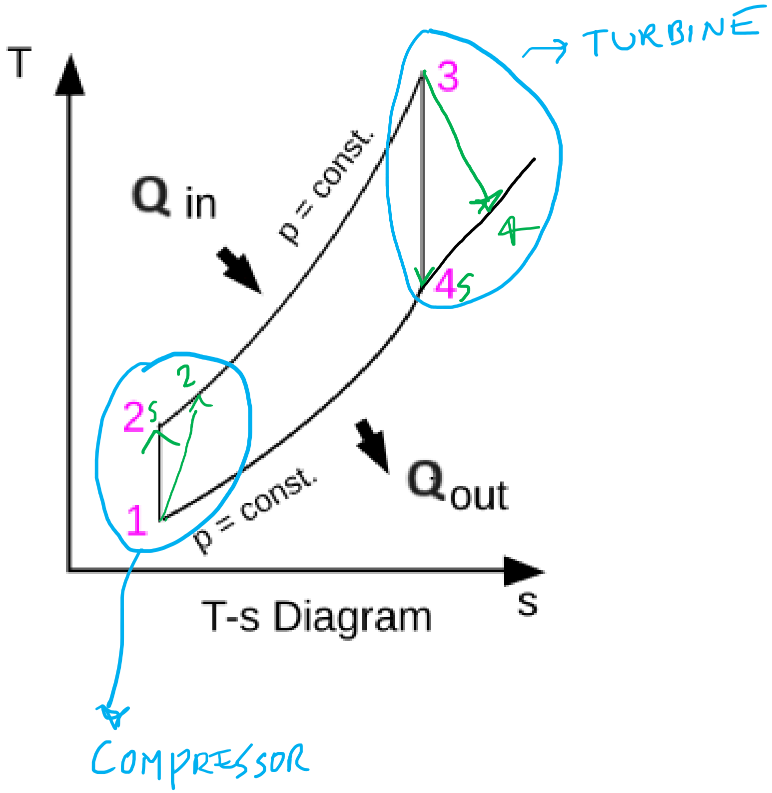


Fig. 9.8 Simple gas turbine. (*a*) Open to the atmosphere. (*b*) Closed.



ISENTROPIC EFFICIENCIES OF STEADY-FLOW DEVICES

$(s_1 = s_2)$

I wish as an engineer:

-Maximum work of the turbine
- -Pure substance (water, misture LV)
- - **Ideal gas**

-Minimum pump work
- -Pure substance (incompressible fluid)

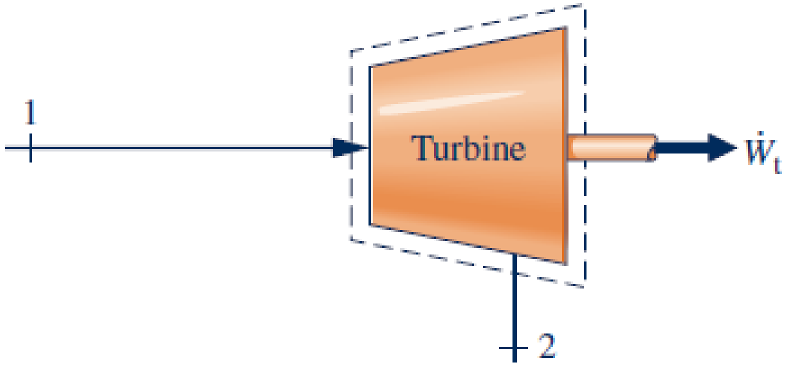
-Minimum work of the compressor
- -Pure substance (Vapor)
- - **Ideal gas**

HOW IS THE ENERGY BALANCE IN A TURBINE??

Adiabatic

$$\cancel{\frac{dE}{dt}} = \cancel{\dot{Q}} - \dot{W} + \sum \dot{m}_{in}h_{in} - \sum \dot{m}_{out}h_{out}$$

Steady state



- Neglecting the changes in potential and kinetic energy
- Assuming steady state.
- Assuming that a process in a turbine occurs adiabatically.

$$\dot{W} = \dot{m}(h_1 - h_2\ ) \rightarrow \frac{\dot{W}}{\dot{m}} = h_1 - h_2$$

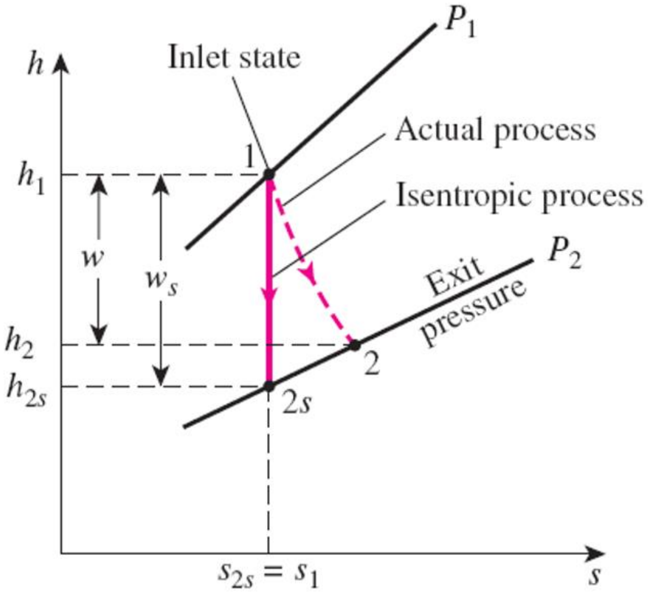
[kJ/ s]

[kJ/ kg]

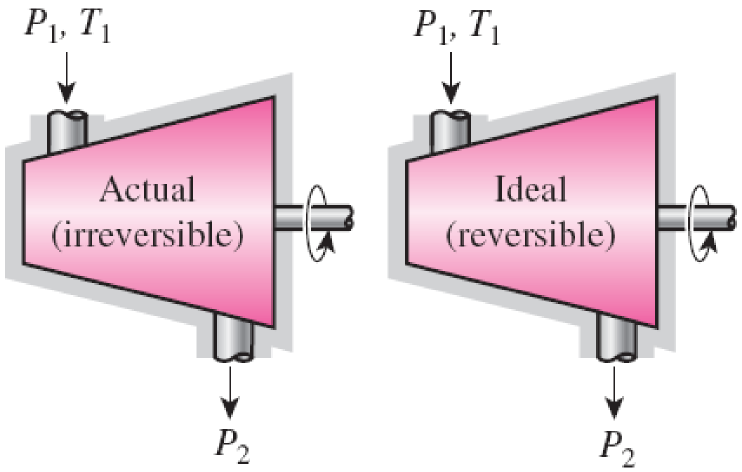
It is the same for compressor assuming the same considerations!!

ISENTROPIC EFFICIENCIES OF STEADY-FLOW DEVICES_TURBINES

$$w = (h_1 - h_2) \;\ldots\ldots\; [\text{ kJ/ kg }]$$



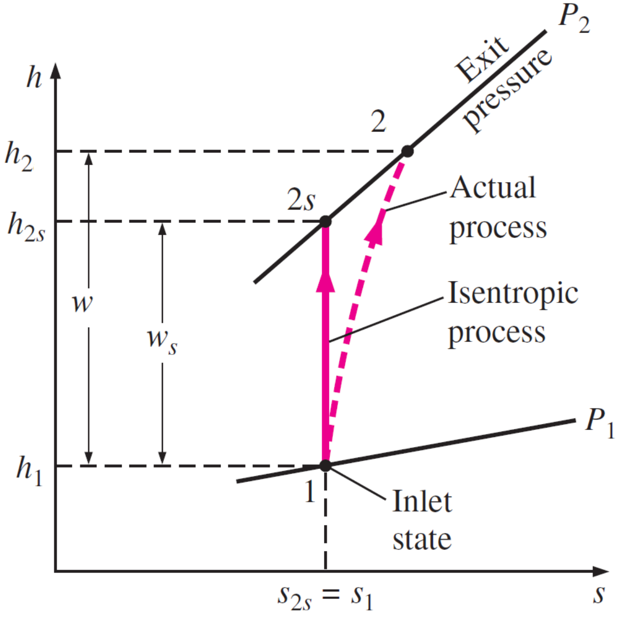
The *h-s* diagram for the actual and isentropic processes of an adiabatic turbine.

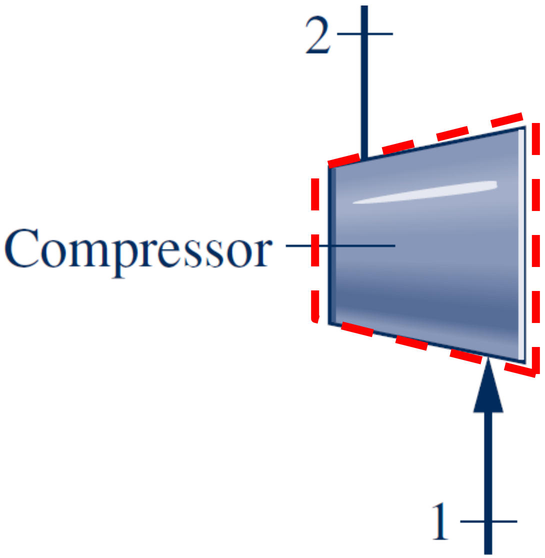


$$\eta_T = \frac{\text{Actual turbine work}}{\text{Isentropic turbine work}} = \frac{w}{w_s}$$

$$\eta_T \cong \frac{h_1 - h_2}{h_1 - h_{2s}} + \text{Eq. aux} \begin{cases} \cdot\ C_p \text{ variable ("}P_r\text{")} \\ \cdot\ C_p \text{ cte ("}K\text{")} \end{cases}$$

Isentropic Efficiencies of Compressor





$$w = (h_1 - h_2) \;.....\; [\,\text{kJ/ kg}\,]$$

$$\eta_C = \frac{\text{Isentropic compressor work}}{\text{Actual compressor work}} = \frac{w_s}{w}$$

$$\eta_C \cong \frac{\boxed{h_1 - h_{2s}}}{h_1 - h_2}$$

+ Eq aux

- Cp variable ("Pr")
- Cp cte ("K")

What happens to ideal gases?

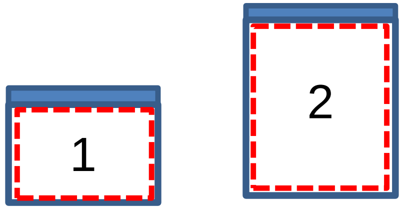
$(s_2 - s_1)\left[\frac{KJ}{Kg.K}\right]$

- **Entropy Change of an Incompressible Substance**
  - Water/Refrigerants – use tables like u and h
- **Entropy Change of an Ideal Gas**

$\hookrightarrow$???

• Ideal gas

$$\underbrace{m(s_2-s_1)}_{\text{Entropy change}} = \int \frac{\delta Q}{T_b} + \sigma$$



- Tables A-22 and A-23 give s° for temperature (Cp Variable)

$\frac{kJ}{kg \cdot K}$

$$\boxed{s_2 - s_1 = s_2^o - s_1^o - R\ln\frac{p_2}{p_1}}$$

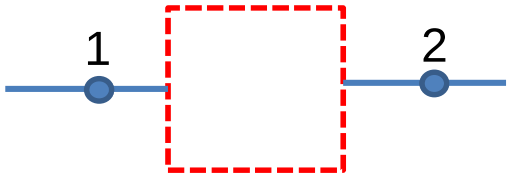
$$\boxed{\bar{s}_2 - \bar{s}_1 = \bar{s}_2^o - \bar{s}_1^o - \bar{R}\ln\frac{p_2}{p_1}}$$

$\frac{kJ}{kmol \cdot K}$

- Constant specific heat (e.g. helium, neon, or argon) (Cp cte)

$$\boxed{s_2 - s_1 = c_p\ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1}}$$

$$\boxed{s_2 - s_1 = c_v\ln\frac{T_2}{T_1} + R\ln\frac{v_2}{v_1}} \rightarrow \frac{kJ}{kg \cdot K}$$



$$\frac{ds}{dt} = \underbrace{\dot{m}(s_2-s_1)}_{\text{Entropy change}} + \int \frac{\delta \dot{Q}}{T_b} + \dot{\sigma}$$

Air

Method 1: Pr

0 isentropico

$$\overset{s_2}{s(T_2, p_2)} - \overset{s_1}{s(T_1, p_1)} = s^\circ(T_2) - s^\circ(T_1) - R \ln \frac{p_2}{p_1} \qquad \text{(6.20a)}$$

$$\frac{p_2}{p_1} = \frac{\exp[s^\circ(T_{2s})/R]}{\exp[s^\circ(T_1)/R]} = \frac{p_{r2}\,(s)}{p_{r1}} \qquad (s_1 = s_2, \text{ air only})$$

$P_{r(2_s)}$

$$\frac{p_2}{p_1} = \frac{p_{r2}}{p_{r1}} \qquad \text{(6.41)}$$

$s_1 = s_2$ (air only), p_r and v_r from Table A-22

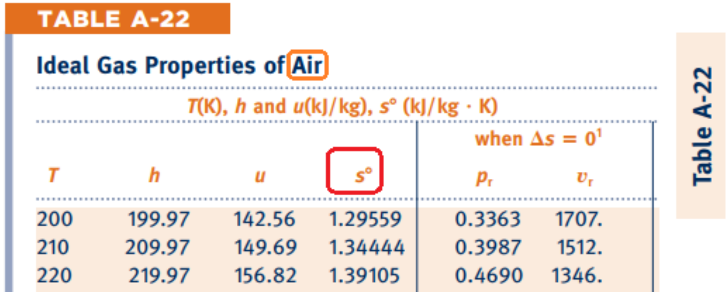


TABLE A-22

Ideal Gas Properties of Air

T(K), h and u(kJ/kg), s° (kJ/kg · K)

				when $\Delta s = 0$[1]	
T	h	u	s°	p_r	v_r
200	199.97	142.56	1.29559	0.3363	1707.
210	209.97	149.69	1.34444	0.3987	1512.
220	219.97	156.82	1.39105	0.4690	1346.

Table A-22

Method 2: Cp constant

0 isentropico

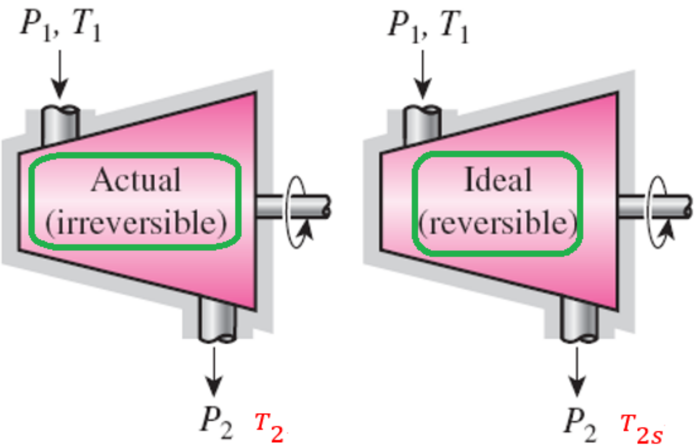
$$\boxed{\overset{s_2}{s(T_2, p_2)} - \overset{s_1}{s(T_1, p_1)} = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}} \qquad \text{(6.22)}$$

$$\text{(6.43)} \qquad \frac{T_{2s}}{T_1} = \left(\frac{p_2}{p_1}\right)^{(k-1)/k}$$

$s_1 = s_2$, constant specific heat ratio k

Method 1: Pr

$$s(T_2, p_2) - s(T_1, p_1) = s^\circ(T_2) - s^\circ(T_1) - R \ln \frac{p_2}{p_1} \quad \text{(6.20a)}$$

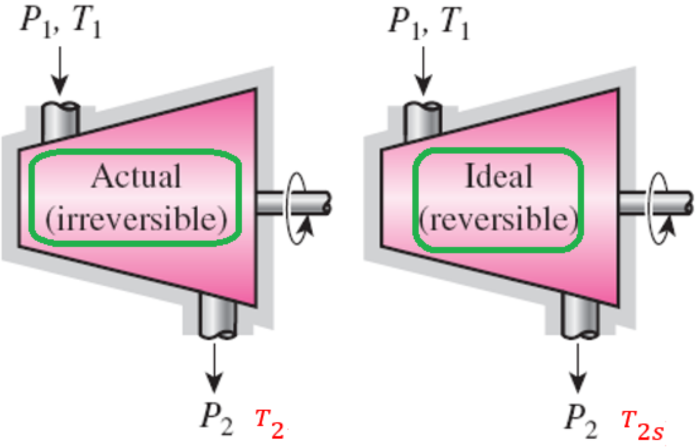


$$\eta_T \cong \frac{\overset{T_1}{h_1} - \overset{T_2}{h_2}}{h_1 - \underset{T_{2s}}{h_{2s}}} \qquad (1)$$

$$\frac{p_2}{p_1} = \frac{\overset{T_{2s}}{p_{r2s}}}{\underset{T_1}{p_{r1}}} \qquad (2)$$

Method 2: Cp constant

$$\boxed{s(T_2, p_2) - s(T_1, p_1) = c_p \ln\frac{T_2}{T_1} - R \ln\frac{p_2}{p_1}} \qquad (6.22)$$



$$\eta_T = \frac{\dot{W}_a}{\dot{W}_s} = \frac{\cancel{\dot{m}}c_p(T_1 - T_2)}{\cancel{\dot{m}}c_p(T_1 - \boldsymbol{T_{2s}})} \qquad (1)$$

$$\frac{T_{2s}}{T_1} = \left(\frac{p_2}{p_1}\right)^{(k-1)/k} \qquad (2)$$

EXAMPLE 4 – CYCLE

The figure provides the schematic of a heat pump using Refrigerant 134a as the working fluid, together with steady-state data at key points. The mass flow rate of the refrigerant is 6.999 kg/min, and the power input to the adiabatic compressor is 5.17 kW. The valve is adiabatic. (a) The isentropic efficiency of the compressor (b) Determine the coefficient of performance for the heat pump. (c) The rate of entropy generation in each equipment $[\frac{kW}{K}]$ (d) The rate of entropy generation of the cycle $[\frac{kW}{K}]$, assuming that in the condenser **(TH)** =20°C and in the evaporator **(TL)** =1°C e) T[(°C)]-s[kJ/kg.K] diagram.