Thermodynamic_2026_1

Second Law_S14

ISENTROPIC EFFICIENCIES_ Ideal gas

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Thermodynamics _ Evaluation

COMPONENTE	%
EXAMEN 1	20
EXAMEN 2	30
L	20
C	30
APROBACIÓN: 11/20	

EVALUACIÓN CONTINUA (C) :

SEMANA 1	QUIZ VIRTUAL	SE CALIFICA AUTOMATICO NO NECESITA PROCEDIMIENTO
SEMANA 2	QUIZ VIRTUAL	
SEMANA 3	QUIZ VIRTUAL	.
SEMANA 4	**GRADESCOPE**+QUIZ AULA	GRADESCOPE: Esta parte incluye subir un archivo con la solución QUIZ AULA : SEMANAS 1-2
SEMANA 5	QUIZ VIRTUAL+ QUIZ AULA	QUIZ AULA : SEMANAS 1-3
SEMANA 6	**QUIZ VIRTUAL+GRADESCOPE**	
SEMANA 7	QUIZ VIRTUAL	
SEMANA 8	**GRADESCOPE**+ QUIZ AULA	QUIZ AULA : SEMANAS 1-6
SEMANA 9		
SEMANA 10	**EXAMEN 1**+QUIZ VIRTUAL	
SEMANA 11	QUIZ VIRTUAL	
SEMANA 12	QUIZ VIRTUAL+ QUIZ AULA	QUIZ AULA : SEMANAS 1-8
SEMANA 13	**QUIZ VIRTUAL+GRADESCOPE**	QUIZ AULA : SEMANAS 1-11
SEMANA 14	QUIZ VIRTUAL+QUIZ AULA	QUIZ AULA : SEMANAS 1-13
SEMANA 15		
SEMANA 16	**EXAMEN 2**	

***LOS QUIZZES VIRTUALES O LOS GRADESCOPE (TAREA INDIVIDUAL) se abren lunes 2pm y se cierran sábado media noche. El quizz virtual se presenta hasta un máximo de 3 veces.**

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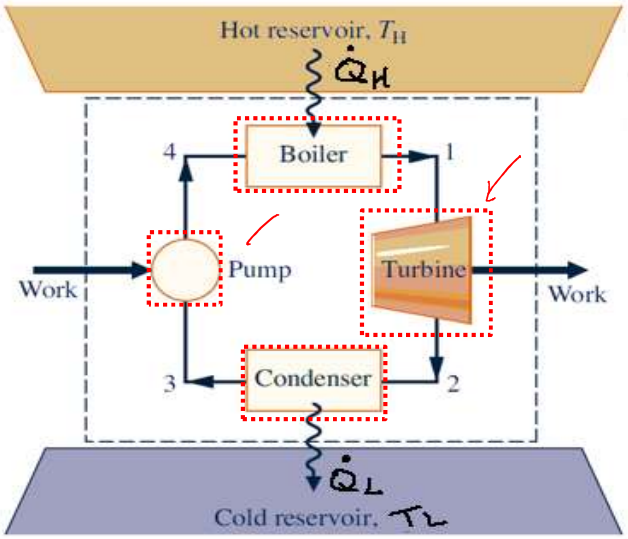
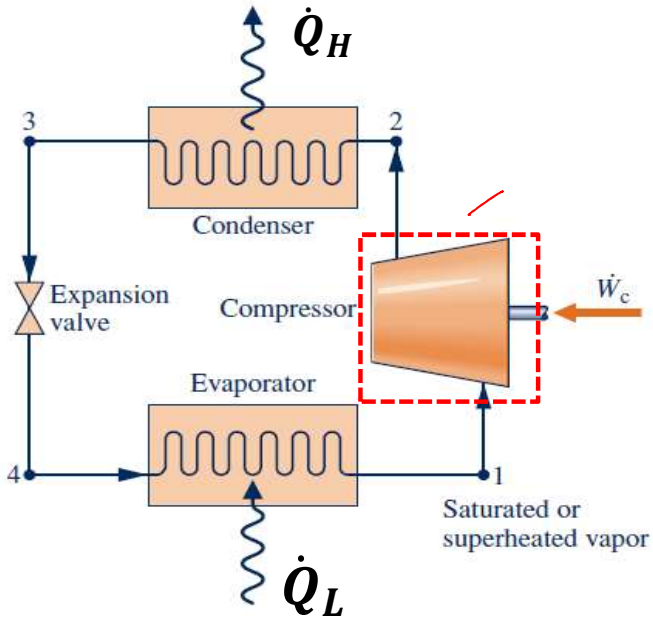
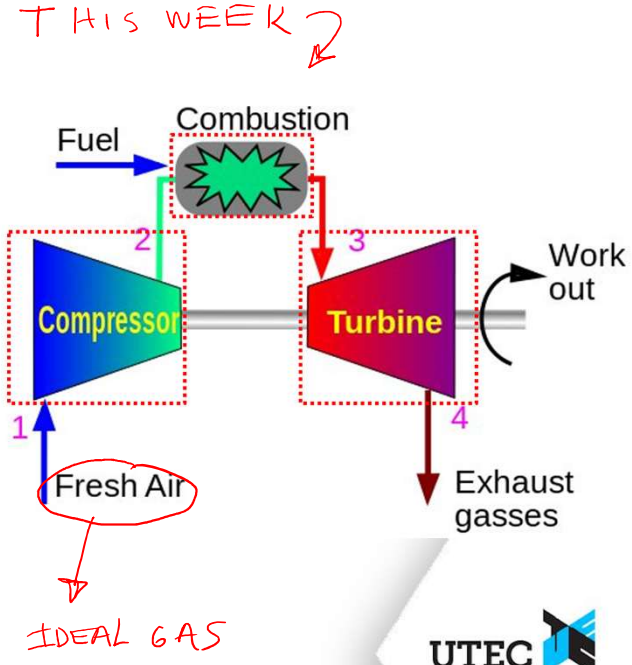
OBJECTIVE

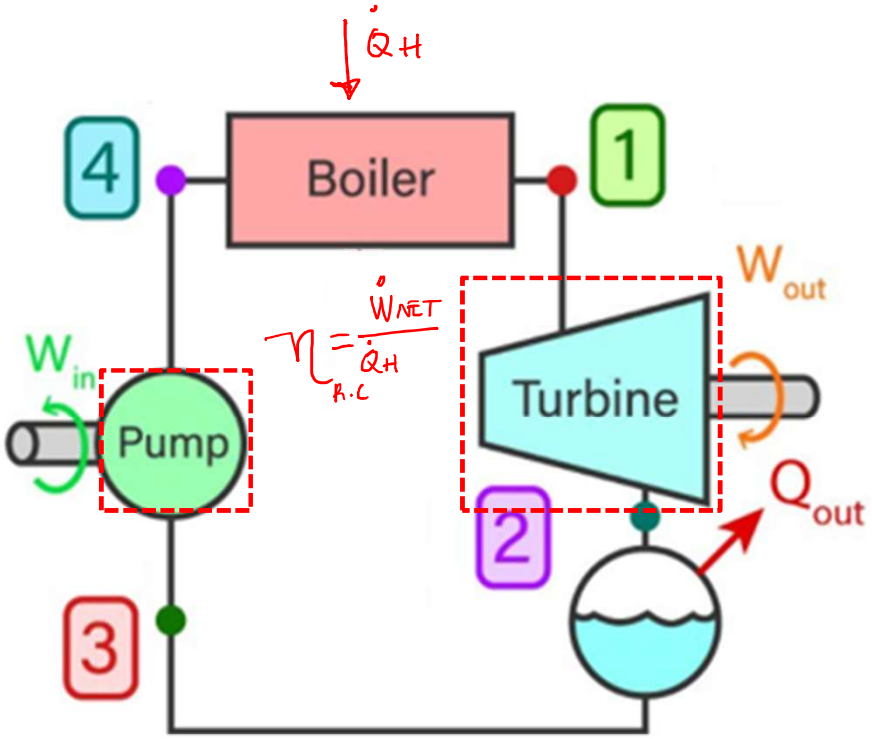
Session 27-28: Isentropic efficiency for turbines and compressors using ideal gases.

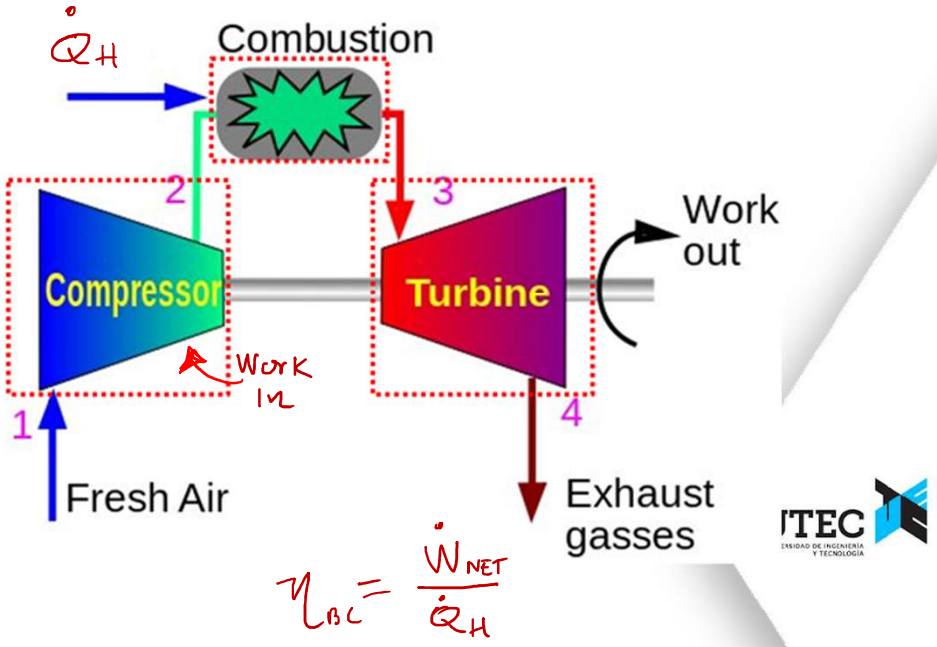
Why do we want to study the isentropic efficiency of turbines, compressors and pumps?



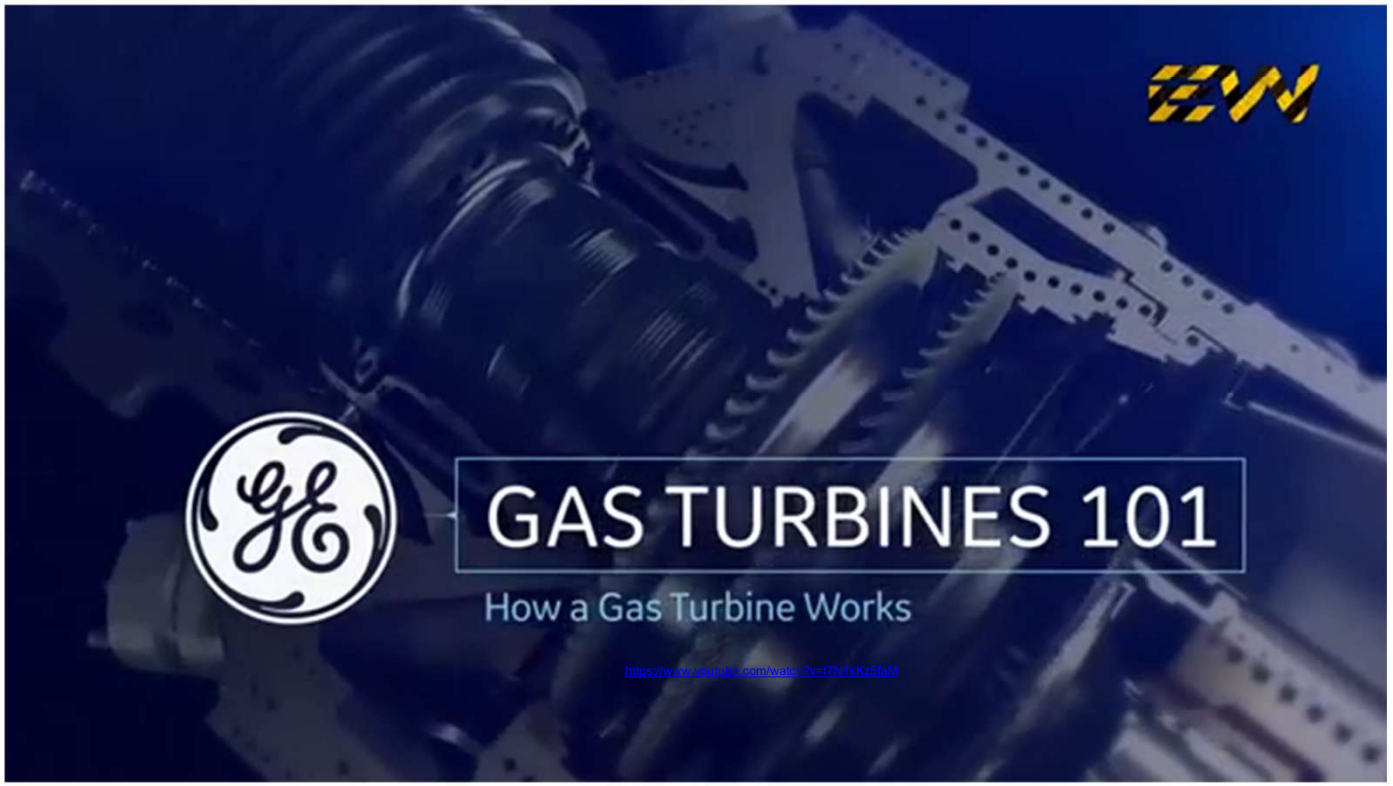








Both are used to produce power, which one is better?



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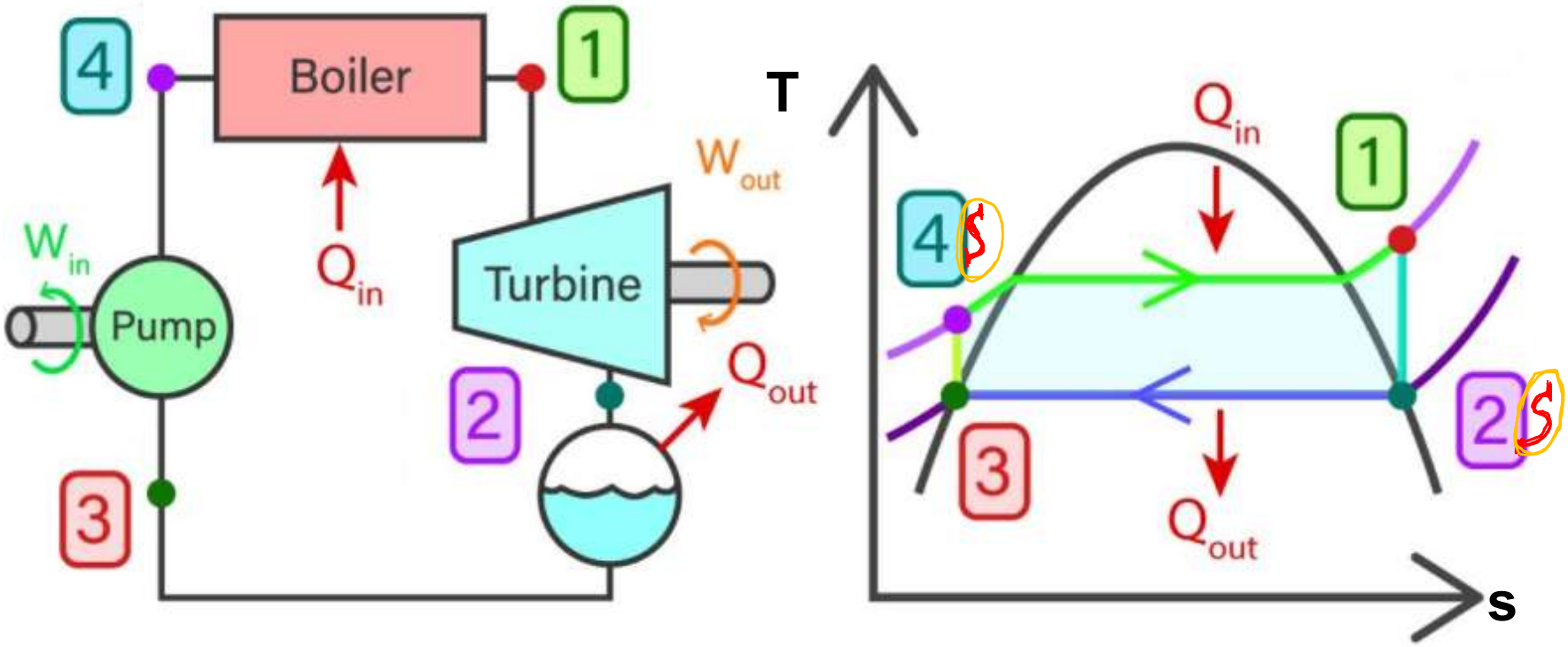
THE SIMPLE IDEAL RANKINE CYCLE_ Heat Engine with LV substance



https://www.youtube.com/watch?v=21XFZaxity8

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IDEAL BRAYTON CYCLE

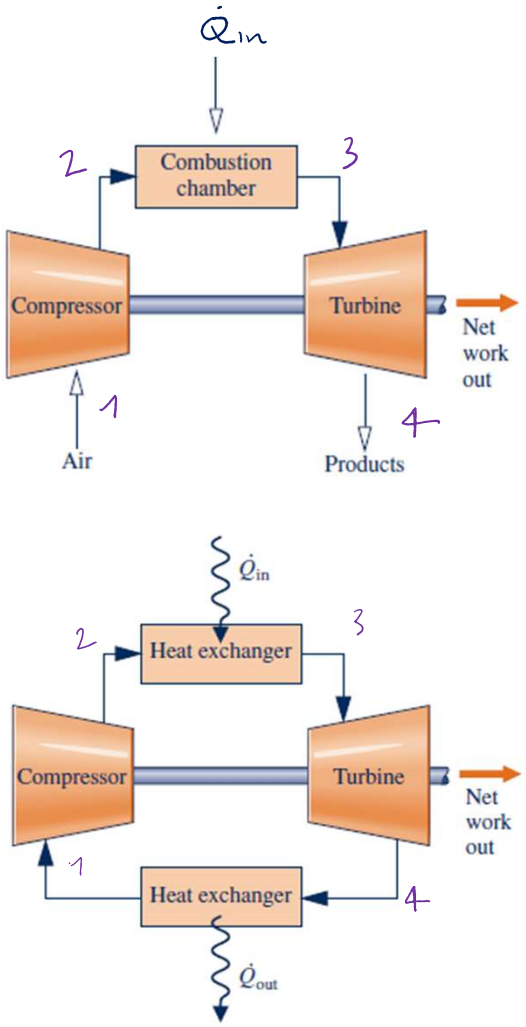
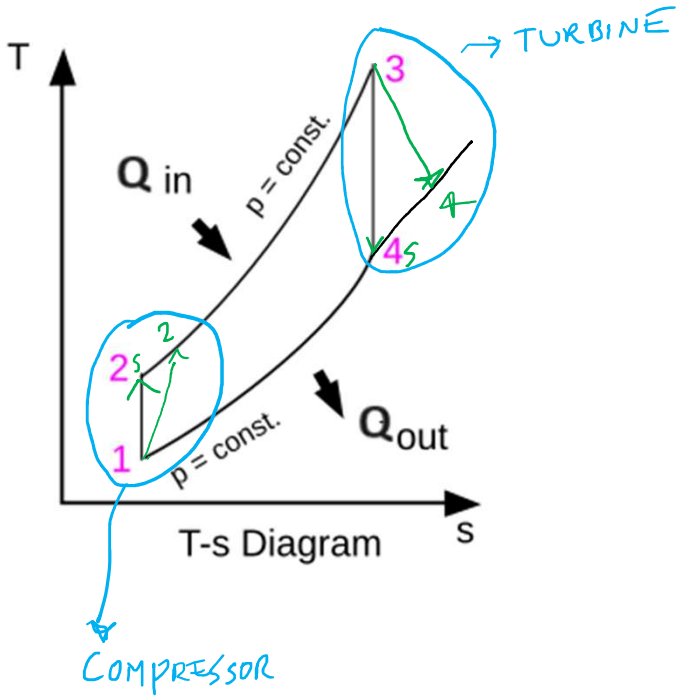


Fig. 9.8 Simple gas turbine. (*a*) Open to the atmosphere. (*b*) Closed.



ISENTROPIC EFFICIENCIES OF STEADY-FLOW DEVICES

$(s_1 = s_2)$

I wish as an engineer:

-Maximum work of the turbine
- -Pure substance (water, misture LV)
- Ideal gas

-Minimum pump work
- -Pure substance (incompressible fluid)

-Minimum work of the compressor
- -Pure substance (Vapor)
- Ideal gas

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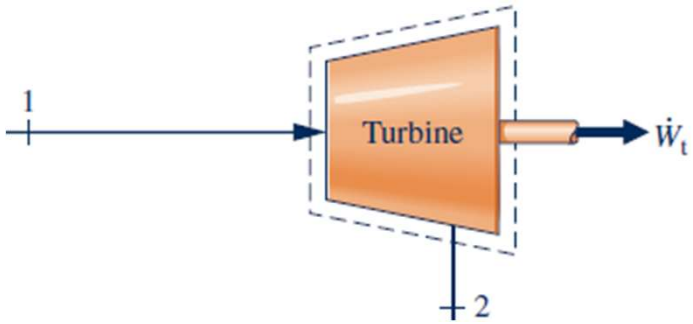
HOW IS THE ENERGY BALANCE IN A TURBINE??

Adiabatic

$$\frac{dE}{dt} = \dot{Q} - \dot{W} + \sum \dot{m}_{in} h_{in} - \sum \dot{m}_{out} h_{out}$$

Steady state



- Neglecting the changes in potential and kinetic energy
- Assuming steady state.
- Assuming that a process in a turbine occurs adiabatically.

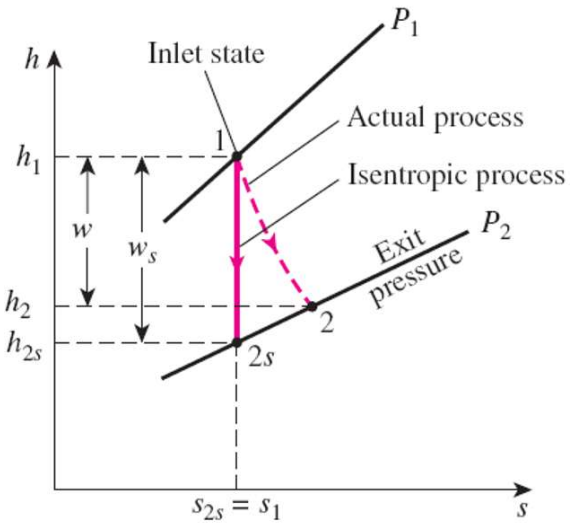
$$\dot{W} = \dot{m}(h_1 - h_2\ ) \rightarrow \quad \frac{\dot{W}}{\dot{m}} = h_1 - h_2$$

[kJ/ s] [kJ/ kg]

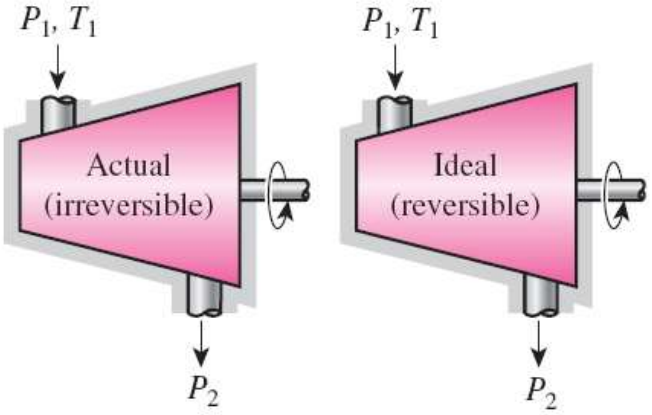
It is the same for compressor assuming the same considerations!!

ISENTROPIC EFFICIENCIES OF STEADY-FLOW DEVICES_TURBINES

$$w = (h_1 - h_2) \quad \ldots\ldots \quad [\text{ kJ/ kg }]$$



The *h-s* diagram for the actual and isentropic processes of an adiabatic turbine.



$$\eta_T = \frac{\text{Actual turbine work}}{\text{Isentropic turbine work}} = \frac{w}{w_s}$$

$$\eta_T \cong \frac{h_1 - h_2}{h_1 - h_{2s}} \quad + \quad \text{Eq. aux} \begin{cases} \cdot\ C_p \text{ variable } (\text{"}P_r\text{"}) \\ \cdot\ C_p \text{ cte } (\text{"}K\text{"}) \end{cases}$$

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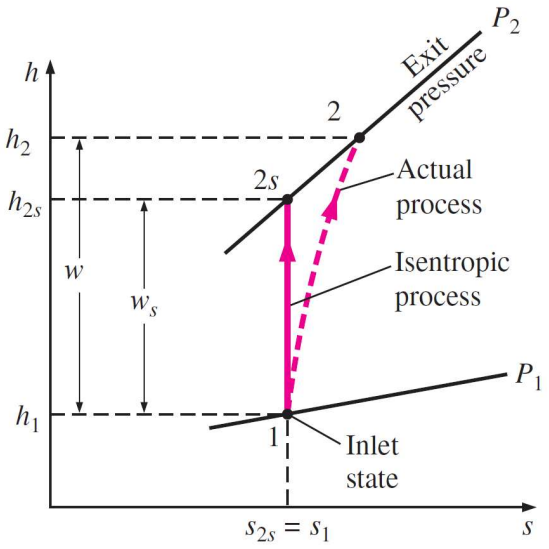
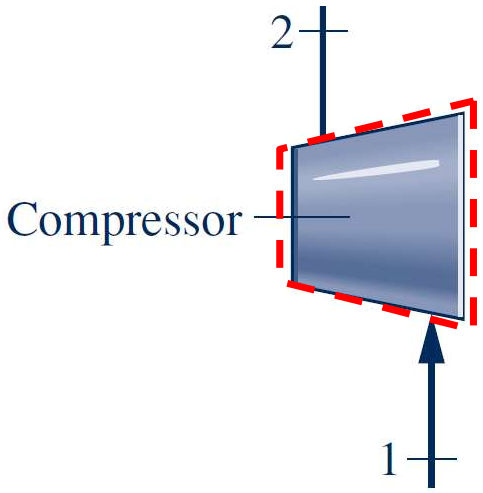
Isentropic Efficiencies of Compressor





$$w = (h_1 - h_2) \ldots\ldots \quad [\text{ kJ/ kg }]$$

$$\eta_C = \frac{\text{Isentropic compressor work}}{\text{Actual compressor work}} = \frac{w_s}{w}$$

$$\eta_C \cong \frac{h_1 - h_{2s}}{h_1 - h_2}$$

+ Eq aux { • Cp variable ("Pr") ; • Cp cte ("K") }

What happens to ideal gases?

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$(s_2 - s_1)\left[\frac{kJ}{kg.K}\right]$

- **Entropy Change of an Incompressible Substance**
  - Water/Refrigerants – use tables like u and h
- **Entropy Change of an Ideal Gas**

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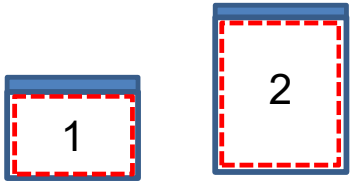
• Ideal gas

$$m(s_2 - s_1) = \int \frac{\delta Q}{T_b} + \sigma$$

Entropy change



- Tables A-22 and A-23 give s° for temperature (c_p Variable)

$$s_2 - s_1 = s_2^o - s_1^o - R\ln\frac{p_2}{p_1} \qquad \frac{kJ}{kg \cdot K}$$

$$\bar{s}_2 - \bar{s}_1 = \bar{s}_2^o - \bar{s}_1^o - \bar{R}\ln\frac{p_2}{p_1} \qquad \frac{kJ}{kmol \cdot K}$$

- Constant specific heat (e.g. helium, neon, or argon) (c_p cte)

$$s_2 - s_1 = c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1}$$

$$s_2 - s_1 = c_v \ln\frac{T_2}{T_1} + R\ln\frac{v_2}{v_1} \qquad \frac{kJ}{kg \cdot K}$$

$$\frac{ds}{dt} = \dot{m}(s_2 - s_1) + \int \frac{\dot{\delta Q}}{T_b} + \dot{\sigma}$$

Entropy change

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Air

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Method 1: Pr

$$s(T_2, p_2) - s(T_1, p_1) = s^\circ(T_2) - s^\circ(T_1) - R \ln \frac{p_2}{p_1} \qquad (6.20a)$$

$$\frac{p_2}{p_1} = \frac{\exp[s^\circ(T_{2s})/R]}{\exp[s^\circ(T_1)/R]} = \frac{p_{r2(2s)}}{p_{r1}} \qquad (s_1 = s_2, \text{ air only})$$

$P_{r(2_s)}$

(6.41) $$\frac{p_2}{p_1} = \frac{p_{r2}}{p_{r1}}$$

$s_1 = s_2$ (air only), p_r and v_r from Table A-22

TABLE A-22

Ideal Gas Properties of Air

T(K), h and u(kJ/kg), s° (kJ/kg · K)

T	h	u	s°	when $\Delta s = 0$: p_r	v_r
200	199.97	142.56	1.29559	0.3363	1707.
210	209.97	149.69	1.34444	0.3987	1512.
220	219.97	156.82	1.39105	0.4690	1346.

Table A-22

Method 2: Cp constant

$$s(T_2, p_2) - s(T_1, p_1) = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} \qquad (6.22)$$

(6.43) $$\frac{T_{2s}}{T_1} = \left(\frac{p_2}{p_1}\right)^{(k-1)/k}$$

$s_1 = s_2$, constant specific heat ratio k

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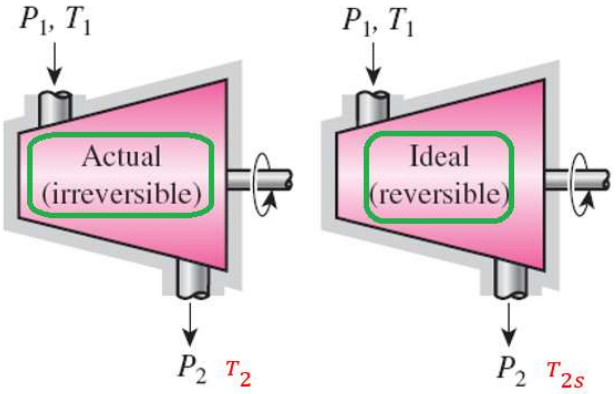
Method 1: Pr

$$s(T_2, p_2) - s(T_1, p_1) = s^\circ(T_2) - s^\circ(T_1) - R \ln \frac{p_2}{p_1} \qquad \text{(6.20a)}$$



$$\eta_T \cong \frac{h_1 - h_2}{h_1 - h_{2s}} \qquad (1)$$

(h_1: T_1; h_2: T_2; h_{2s}: T_{2s})

$$\frac{p_2}{p_1} = \frac{p_{r2s}}{p_{r1}} \qquad (2)$$

(p_{r2s}: T_{2s}; p_{r1}: T_1)

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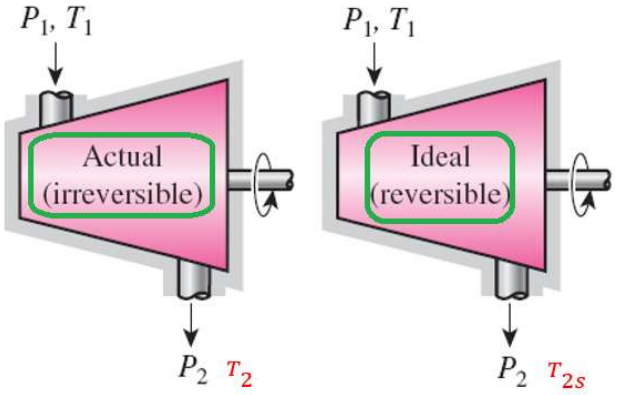
Method 2: Cp constant

$$s(T_2, p_2) - s(T_1, p_1) = c_p \ln\frac{T_2}{T_1} - R \ln\frac{p_2}{p_1} \qquad (6.22)$$



$$\eta_T = \frac{\dot{W}_a}{\dot{W}_s} = \frac{\dot{m}c_p(T_1 - T_2)}{\dot{m}c_p(T_1 - \boldsymbol{T_{2s}})} \qquad (1)$$

$$\frac{T_{2s}}{T_1} = \left(\frac{p_2}{p_1}\right)^{(k-1)/k} \qquad (2)$$

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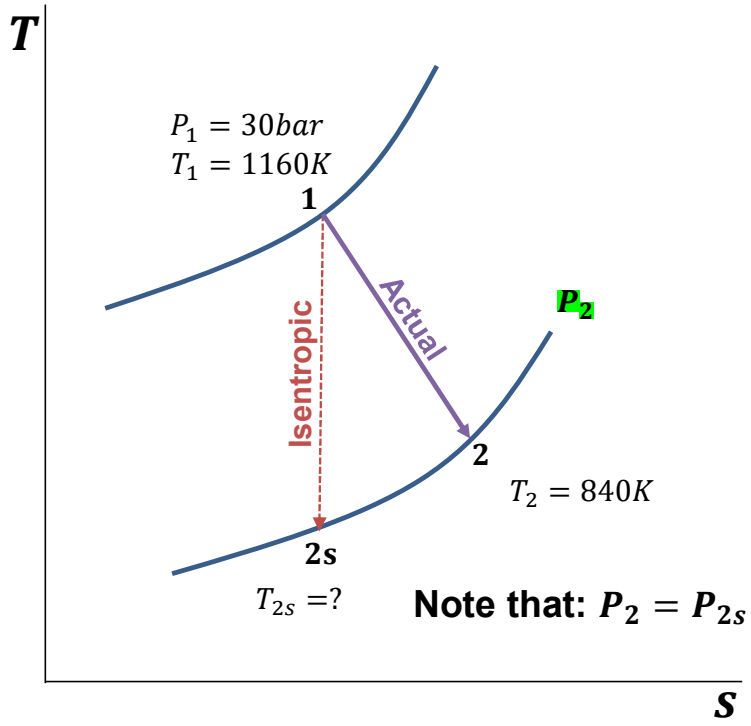
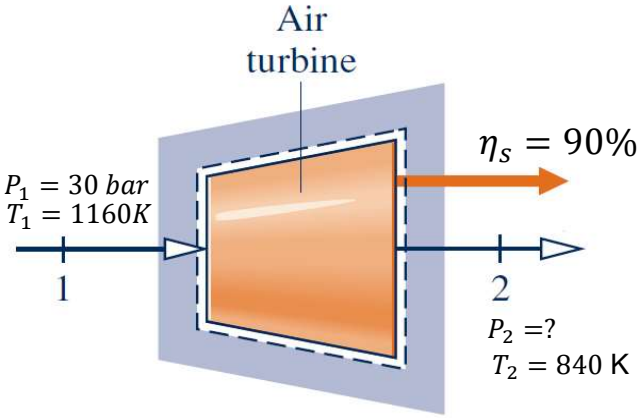
Example 1: Air at 1160 K, 30 bar enters a turbine operating at steady state and expands adiabatically to the exit, where the temperature is 840 K. If the isentropic turbine efficiency is 90%, determine (a) the pressure at the exit, in bar, and (b) the work developed, in kJ per kg of air flowing. Assume ideal gas behavior for the air and ignore kinetic and potential energy effects. USE BOTH METHODS.





We know that:

- Isentropic efficiency = 90%
- Steady state
- Potential and kinetic energy are negligible
- Air as ideal gas

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Method 1: Pr

Method 1: Pr

Isentropic Efficiency:

$$\eta_s = \frac{h_1 - h_2}{h_1 - h_{2s}} \longrightarrow 0.9 = \frac{1230.92 - 866.08}{1230.92 - h_{2s}} \longrightarrow h_{2s} = 825.5422\ kJ/kg$$

TABLE A-22

(Continued)

T(K), h and u(kJ/kg), s° (kJ/kg · K)

T	h	u	s°	when $\Delta s = 0^\dagger$ p_r	v_r
750	767.29	551.99	2.64737	37.35	57.63
760	778.18	560.01	2.66176	39.27	55.54
770	789.11	568.07	2.67595	41.31	53.39
780	800.03	576.12	2.69013	43.35	51.64
790	810.99	584.21	2.70400	45.55	49.86
800	821.95	592.30	2.71787	47.75	48.08
820	843.98	608.59	2.74504	52.59	44.84
840	866.08	624.95	2.77170	57.60	41.85
860	888.27	641.40	2.79783	63.09	39.12
880	910.56	657.95	2.82344	68.98	36.61
900	932.93	674.58	2.84856	75.29	34.31
920	955.38	691.28	2.87324	82.05	32.18
940	977.92	708.08	2.89748	89.28	30.22
960	1000.55	725.02	2.92128	97.00	28.40
980	1023.25	741.98	2.94468	105.2	26.73
1000	1046.04	758.94	2.96770	114.0	25.17

T	h	u	s°	when $\Delta s = 0$ p_r	v_r
980	1023.25	741.98	2.94468	105.2	26.73
1000	1046.04	758.94	2.96770	114.0	25.17
1020	1068.89	776.10	2.99034	123.4	23.72
1040	1091.85	793.36	3.01260	133.3	22.39
1060	1114.86	810.62	3.03449	143.9	21.14
1080	1137.89	827.88	3.05608	155.2	19.98
1100	1161.07	845.33	3.07732	167.1	18.896
1120	1184.28	862.79	3.09825	179.7	17.886
1140	1207.57	880.35	3.11883	193.1	16.946
1160	1230.92	897.91	3.13916	207.2	16.064
1180	1254.34	915.57	3.15916	222.2	15.241
1200	1277.79	933.33	3.17888	238.0	14.470
1220	1301.31	951.09	3.19834	254.7	13.747
1240	1324.93	968.95	3.21751	272.3	13.069
1260	1348.55	986.90	3.23638	290.8	12.435
1280	1372.24	1004.76	3.25510	310.4	11.835

T_1 Pr_1 T_1

(b) The actual work is :

$$\frac{\dot{W}_a}{\dot{m}} = h_1 - h_2$$

$$\frac{\dot{W}_a}{\dot{m}} = 1230.92 - 866.08$$

$$\boldsymbol{\frac{\dot{W}_a}{\dot{m}} = 364.84\ kJ/kg}$$

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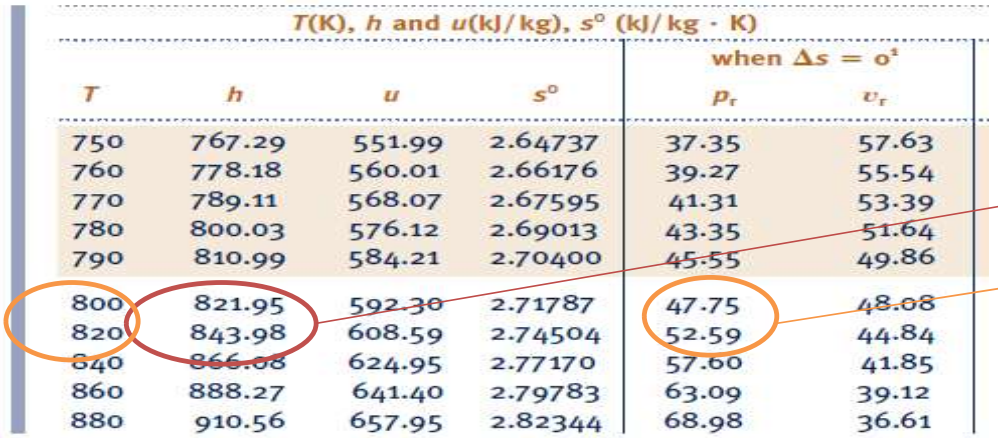
$a)$ Finding the isentropic temperature from h_{2s}:



T(K), h and u(kJ/kg), s^o (kJ/kg · K)				when $\Delta s = o^1$	
T	h	u	s^o	p_r	v_r
750	767.29	551.99	2.64737	37.35	57.63
760	778.18	560.01	2.66176	39.27	55.54
770	789.11	568.07	2.67595	41.31	53.39
780	800.03	576.12	2.69013	43.35	51.64
790	810.99	584.21	2.70400	45.55	49.86
800	821.95	592.30	2.71787	47.75	48.08
820	843.98	608.59	2.74504	52.59	44.84
840	866.08	624.95	2.77170	57.60	41.85
860	888.27	641.40	2.79783	63.09	39.12
880	910.56	657.95	2.82344	68.98	36.61

$h_{2s} = 825.5422\ kJ/kg$

After interpolating:

$T_{2s} = 803.2612\ K$

$Pr_{(2s)} = 48.5392$

The pressure can be calculated from:

$$\frac{P_2}{P_1} = \frac{P_{r(2s)}}{P_{r(1)}} \longrightarrow \frac{P_2}{30} = \frac{48.5392}{207.2}$$

$$\boldsymbol{P_2 = 7.0279\ bar}$$

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HOMEWORK: The rate of entropy generation [kJ/kg.K] is:

$$\frac{ds}{dt} = \dot{m}(s_1 - s_2) + \frac{\dot{Q}}{T_0} + \dot{\sigma}$$

$$\frac{\dot{\sigma}}{\dot{m}} = \sigma = s_2 - s_1 = \begin{cases} = s_2^0 - s_1^0 - R \ln \dfrac{P_2}{P_1} \\ \\ = c_p \ln \dfrac{T_2}{T_1} - R \ln \dfrac{P_2}{P_1} \end{cases}$$

→ Pr method → c_p variable ✓

→ c_p constante ✗

Method 2: Cp constant

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Method 2: Considering Cp constant

Isentropic Efficiency: $$\eta_s = \frac{\dot{W}_a}{\dot{W}_s} = \frac{\dot{m}c_p(T_1 - T_2)}{\dot{m}c_p(T_1 - \boldsymbol{T_{2s}})}$$

$$\frac{T_{2s}}{T_1} = \left(\frac{\boldsymbol{P_2}}{P_1}\right)^{\frac{k-1}{k}}$$

Substituting isentropic efficiency:

$$\frac{1160 - 840}{1160 - T_{2s}} = 0.9 \longrightarrow \boldsymbol{T_{2s} = 804.4444\ K}$$

TABLE A-20

Ideal Gas Specific Heats of Som

Temp. K	c_p	c_v	k
	Air		
250	1.003	0.716	1.401
300	1.005	0.718	1.400
350	1.008	0.721	1.398
400	1.013	0.726	1.395
450	1.020	0.733	1.391
500	1.029	0.742	1.387
550	1.040	0.753	1.381
600	1.051	0.764	1.376
650	1.063	0.776	1.370
700	1.075	0.788	1.364
750	1.087	0.800	1.359
800	1.099	0.812	1.354
900	1.121	0.834	1.344
1000	1.142	0.855	1.336

The average temperature is (1160+840)/2 = 1000K, we can use $\boldsymbol{c_p}$ an $\boldsymbol{k}$ at 1000K in table A-20.

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Finding the pressure:

$$\frac{T_{2s}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} \longrightarrow \frac{804.44}{1160} = \left(\frac{P_2}{30}\right)^{\frac{1.336-1}{1.336}} \longrightarrow$$

(a) $\boldsymbol{P_2 = 6.9994\ bar}$

TABLE A-20

Ideal Gas Specific Heats of Som

Temp. K	c_p	c_v	k
	Air		
250	1.003	0.716	1.401
300	1.005	0.718	1.400
350	1.008	0.721	1.398
400	1.013	0.726	1.395
450	1.020	0.733	1.391
500	1.029	0.742	1.387
550	1.040	0.753	1.381
600	1.051	0.764	1.376
650	1.063	0.776	1.370
700	1.075	0.788	1.364
750	1.087	0.800	1.359
800	1.099	0.812	1.354
900	1.121	0.834	1.344
1000	1.142	0.855	1.336

$k = 1.336$

Finding the work:

$$\frac{\dot{W}_a}{\dot{m}_a} = (h_1 - h_2)$$

$$\frac{\dot{W}_a}{\dot{m}} = c_p(T_1 - T_2)$$

$$\frac{\dot{W}_a}{\dot{m}} = 1.142\,\frac{kJ}{kgK} \times (1160 - 840)K$$

(b) $\boldsymbol{\frac{\dot{W}_a}{\dot{m}} = 365.44\ \frac{kJ}{kg}}$

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HOMEWORK: The rate of entropy generation [kJ/kg.K] is:

$$\frac{ds}{dt} = \dot{m}(s_1 - s_2) + \frac{\dot{Q}}{T_0} + \dot{\sigma}$$

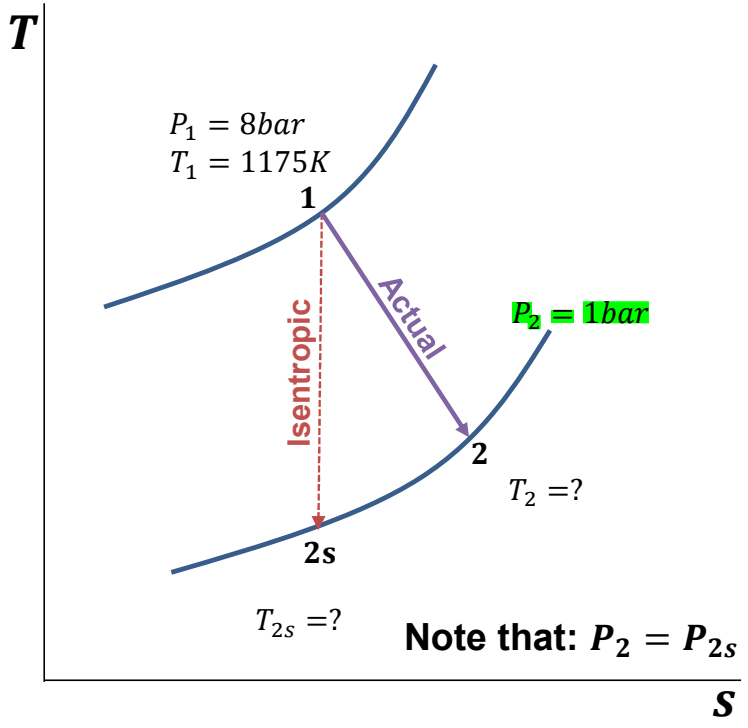
$$\frac{\dot{\sigma}}{\dot{m}} = \sigma = s_2 - s_1 = \begin{cases} = s_2^0 - s_1^0 - R \ln \dfrac{P_2}{P_1} \\ \\ = c_p \ln \dfrac{T_2}{T_1} - R \ln \dfrac{P_2}{P_1} \end{cases}$$

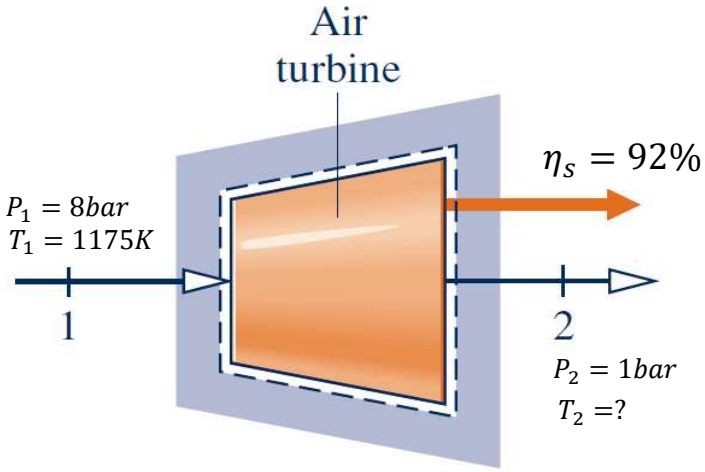
→ Pr method

→ c_p variable ✗

→ c_p constante ✓

Example (TURBINE) Air at 1175 K, 8 bar enters a turbine operating at steady state and expands adiabatically to 1 bar. The isentropic turbine efficiency is 92%. Employing the ideal gas model with k = 1.34 for the air, determine (a) the work developed by the turbine, in kJ per kg of air flowing, and (b) the temperature at the exit, in K. Ignore kinetic and potential energy effects. **USE THE TWO METHODS**





We know that:

- Isentropic efficiency = 92%
- Steady state
- Potential and kinetic energy are negligible
- Air as ideal gas

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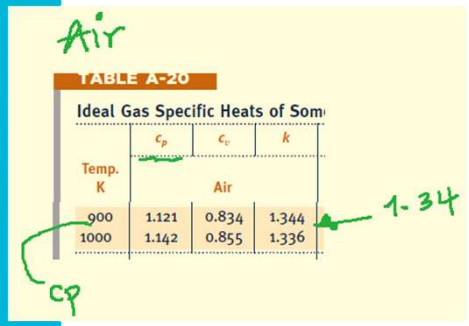
Method 1: Considering Cp constant

The problem suggests to use $k = 1.34$, so we can find isentropic temperature T_{2s} and c_p

$$\frac{T_{2s}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} \longrightarrow \frac{T_{2s}}{1175} = \left(\frac{1}{8}\right)^{\frac{1.34-1}{1.34}} \longrightarrow \boldsymbol{T_{2s} = 693.2592\ K}$$

Now we can estimate the actual temperature from entropy efficiency:

$$\eta_s = \frac{\boldsymbol{T_1 - T_2}}{\boldsymbol{T_1 - T_{2s}}} = 0.92 \longrightarrow \frac{1175 - T_2}{1175 - 693.26} = 0.92 \longrightarrow \boldsymbol{T_2 = 731.7985\ K}$$



Interpolating with k=1.34:

$$c_p = 1.1315 \frac{kJ}{kgK}$$

And the turbine work is:

$$\frac{\dot{W}_a}{\dot{m}} = h_1 - h_2 = \boldsymbol{c_p(T_1 - T_2)} \longrightarrow \frac{\dot{W}_a}{\dot{m}} = 1.131\ \frac{kJ}{kgK}(1175 - 731.7985) \longrightarrow \boldsymbol{\frac{\dot{W}_a}{\dot{m}} = 501.4825 \frac{kJ}{kg}}$$

Air

TABLE A-20

Ideal Gas Specific Heats of Som

Temp. K	c_p	c_v	k
	Air		
900	1.121	0.834	1.344
1000	1.142	0.855	1.336

1.34

cp

```python
import numpy as np

k = 1.34  # valor de k donde queremos cp

# Datos ordenados por k (de menor a mayor)
k_tab  = np.array([1.336, 1.344])   # k
cp_tab = np.array([1.142, 1.121])   # cp correspondiente (kJ/kg·K)

cp = np.interp(k, k_tab, cp_tab)

print(f"cp interpolado = {cp:.4f} kJ/kg·K")
```

```
cp interpolado = 1.1315 kJ/kg·K
```

Alternative way to find cp (ideal gases):

$$k = \frac{c_p}{c_v} = \frac{c_p}{c_p - R} \longrightarrow c_p = R\left(\frac{k}{k-1}\right) \longrightarrow c_p = 0.287\frac{kJ}{kgK}\left(\frac{1.34}{1.34-1}\right)$$

$$c_p = 1.1311\frac{kJ}{kgK}$$

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Method 2: Ideal gas tables

$\eta = \frac{h_1 - h_2}{h_1 - h_{2s}}$?

$\frac{P_2}{P_1} = \frac{P_{r2s}}{P_{r1}}$ ⇒

(Continued)

T(K), h and u(kJ/kg), s° (kJ/kg · K)

				when $\Delta s = 0^1$	
T	h	u	s°	p_r	v_r
1160	1230.92	897.91	3.13916	207.2	16.064
1180	1254.34	915.57	3.15916	222.2	15.241

$h_1 = 1248.485 \frac{KJ}{Kg}$

We can calculate $P_{r(2s)}$ with pressures and T1 :

$$\frac{P_2}{P_1} = \frac{P_{r(2s)}}{P_{r(1)}} \longrightarrow \frac{1}{8} = \frac{P_{r(2s)}}{218.45}$$

Interpolating at T_1=1175K

$$P_{r(2s)} = 27.3062$$

```
T_tab1  = np.array([1160.0, 1180])      # K  (ejemplo)
h_tab1  = np.array([ 1230.92 , 1254.34 ])         # kJ/kg  (pon tus valores)
Pr_tab1 = np.array([ 207.2, 222.2])         # adim.  (pon tus valores)

h1  = np.interp(T1, T_tab1, h_tab1)
Pr1 = np.interp(T1, T_tab1, Pr_tab1)
h1 = round(h1,4)
Pr1 = round(Pr1,4)
# ========================
# 2. Cálculo de Pr(2s)
# ========================
Pr2s = (P2 / P1) * Pr1
Pr2s = round(Pr2s,4)
```

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Finding the air conditions that results in $P_{r(2s)} = 27.3062$:

TABLE A-22

Ideal Gas Properties of Air

T(K), h and u(kJ/kg), $s°$ (kJ/kg · K)

				when $\Delta s = 0$	
T	h	u	$s°$	p_r	v_r
690	702.52	504.45	2.55731	27.29	72.56
700	713.27	512.33	2.57277	28.80	69.76

Interpolating at $P_{r(2s)} = 27.3062$

Interpolating we have:

$$P_{r(2s)} = 27.3062$$

$$T_{2s} = 690.1073\ K$$

$$h_{2s} = 702.6353\ kJ/kg$$

Therefore, the isentropic work is:

$$\dot{W}_s = \dot{m}(h_1 - h_{2s})$$

$$\frac{\dot{W}_s}{\dot{m}} = 1248.485 - 702.6353$$

$$\frac{\dot{W}_s}{\dot{m}} = w_s = 545.8497\ kJ/kg$$

The actual work is:

$$\eta_s = \frac{w_a}{w_s} \rightarrow w_a = 0.92 \times 545.8497\ kJ/kg$$

$$\boldsymbol{w_a = 502.1817\ kJ/kg}$$

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The actual enthalpy at state 2: $\dot{W}_a = \dot{m}(h_1 - h_2)$ $\longrightarrow$ $h_2 = h_1 - \frac{\dot{W}_a}{\dot{m}}$

$$h_2 = 1248.485 - 502.1817$$

$$h_2 = 746.3033\ kJ/kg$$

Finding the air conditions that results in $h_2 = 746.302 kJ/kg$

TABLE A-22

Ideal Gas Properties of Air

T(K), h and u(kJ/kg), s° (kJ/kg · K)

				when $\Delta s = 0$	
T	h	u	s°	p_r	v_r
730	745.62	536.07	2.61803	33.72	62.13
740	756.44	544.02	2.63280	35.50	59.82

Interpolating we have:

Interpolating at $h_2 = 746.3033\ kJ/kg$

$$T_2 = 730.6315\ K$$

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HOMEWORK: The rate of entropy generation [kJ/kg.K] is:

$$\frac{ds}{dt} = \dot{m}(s_1 - s_2) + \frac{\dot{Q}}{T_0} + \dot{\sigma}$$

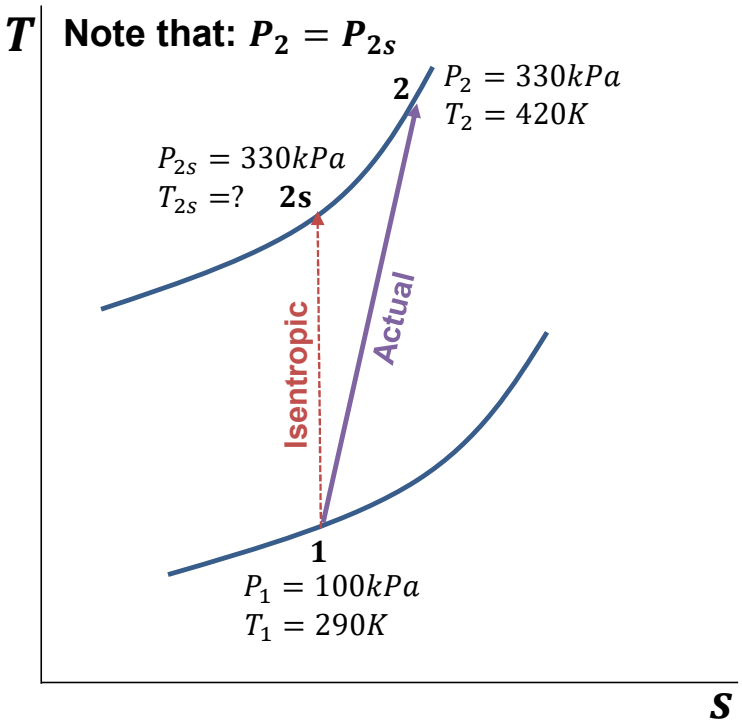
$$\frac{\dot{\sigma}}{\dot{m}} = \sigma = s_2 - s_1 = \begin{cases} = s_2^0 - s_1^0 - R \ln \dfrac{P_2}{P_1} \\ \\ = c_p \ln \dfrac{T_2}{T_1} - R \ln \dfrac{P_2}{P_1} \end{cases}$$

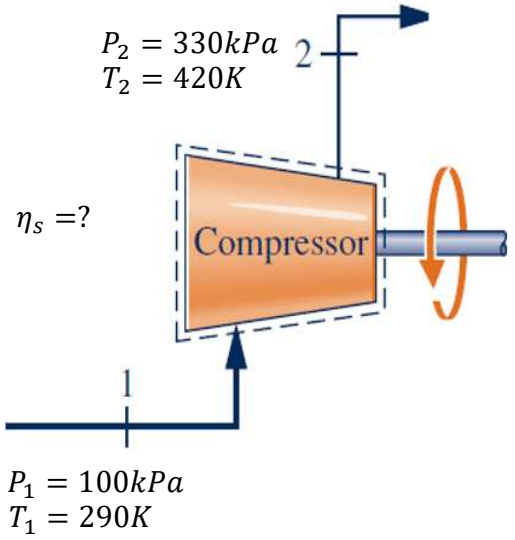
→ Pr method

→ c_p variable

→ c_p constante

Example (COMPRESSOR) Air at 290 K, 100 kPa enters a compressor operating at steady state and is compressed adiabatically to an exit state of 420 K, 330 kPa. The air is modeled as an ideal gas, and kinetic and potential energy effects are negligible. For the compressor, (a) determine the rate of entropy production, in kJ/K per kg of air flowing, and (b) the isentropic compressor efficiency. **USE THE TWO METHODS**.





We know that:

- Steady state
- Potential and kinetic energy are negligible
- Air as ideal gas

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Method 1: Considering Cp constant

We can use c_p and k at T average = 355 K.

Interpolating : $c_p = 1.0085 kJ/kgK, k = 1.3977$

$$\frac{T_{2s}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} \longrightarrow \frac{T_{2s}}{290} = \left(\frac{330}{100}\right)^{\frac{1.3977-1}{1.3977}} \longrightarrow \boldsymbol{T_{2s} = 407.3197K}$$

TABLE A-20

Ideal Gas Specific Heats of Som

Temp. K	c_p	c_v	k
	Air		
250	1.003	0.716	1.401
300	1.005	0.718	1.400
350	1.008	0.721	1.398
400	1.013	0.726	1.395

Finding the Isentropic Efficiency :

$$\eta_s = \frac{T_1 - T_{2s}}{T_1 - T_2} = \frac{290 - 407.32}{290 - 420} = \mathbf{0.9025}$$

$$\boldsymbol{\eta_s = 90.25\%}$$

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The rate of entropy generation (per unit mass) is:

$$\frac{ds}{dt} = \dot{m}(s_1 - s_2) + \frac{\dot{Q}}{T_0} + \dot{\sigma} \longrightarrow \frac{\dot{\sigma}}{\dot{m}} = \sigma = s_2 - s_1 = c_p \ln\frac{T_2}{T_1} - R \ln\frac{P_2}{P_1} \longrightarrow \sigma = 1.0085 \ln\frac{420}{290} - \frac{8.314}{28.97}\ln\frac{330}{100}$$

$$\boldsymbol{\sigma = 0.03088 \frac{kJ}{kgK}}$$

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Method 2: Ideal gas tables

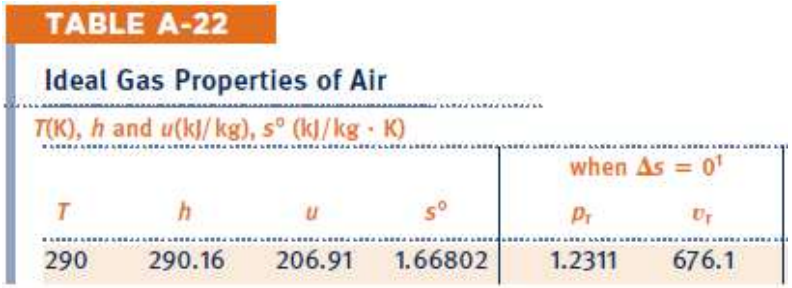

TABLE A-22

Ideal Gas Properties of Air

T(K), h and u(kJ/kg), $s°$ (kJ/kg · K)

				when $\Delta s = 0^1$	
T	h	u	$s°$	p_r	v_r
290	290.16	206.91	1.66802	1.2311	676.1

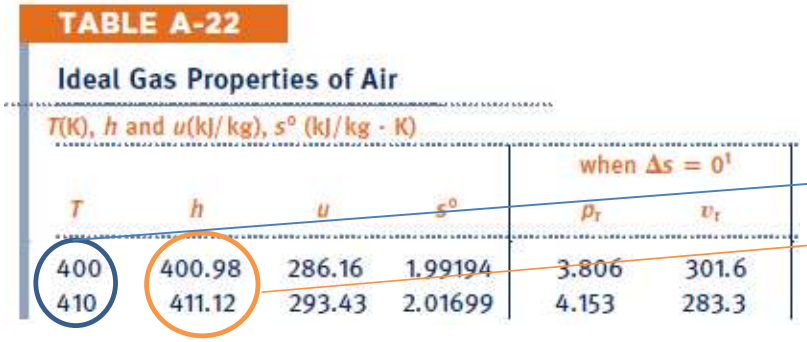

TABLE A-22

Ideal Gas Properties of Air

T(K), h and u(kJ/kg), $s°$ (kJ/kg · K)

				when $\Delta s = 0^1$	
T	h	u	$s°$	p_r	v_r
400	400.98	286.16	1.99194	3.806	301.6
410	411.12	293.43	2.01699	4.153	283.3

We can calculate $P_{r(2s)}$ with pressures and T1 :

$$\frac{P_2}{P_1} = \frac{P_{r(2s)}}{P_{r(1)}} \longrightarrow \frac{330}{100} = \frac{P_{r(2s)}}{1.2311}$$

$$P_{r(2s)} = 4.0626$$

Finding the air conditions that results in $P_{r(2s)} = 4.0626$:

Interpolating we have:

$T_{2s} = 407.3948K$ $\qquad$ $h_{2s} = 408.4783\ kJ/kg$

Isentropic Efficiency:

$$\eta_s = \frac{h_1 - h_{2s}}{h_1 - h_2} \longrightarrow \eta_s = \frac{290.16 - 408.48}{290.16 - 421.26} \longrightarrow \eta_s = 90.25\%$$

TABLE A-22

Ideal Gas Properties of Air

T(K), h and u(kJ/kg), s° (kJ/kg · K)

				when $\Delta s = 0^\dagger$	
T	h	u	s°	p_r	v_r
400	400.98	286.16	1.99194	3.806	301.6
410	411.12	293.43	2.01699	4.153	283.3
420	421.26	300.69	2.04142	4.522	266.6
290	290.16	206.91	1.66802	1.2311	676.1

The rate of entropy generation is:

$$\frac{ds}{dt} = \dot{m}(s_1 - s_2) + \frac{\dot{Q}}{T_0} + \dot{\sigma} \longrightarrow \frac{\dot{\sigma}}{\dot{m}} = \sigma = s_2 - s_1$$

$$\sigma = s_2 - s_1 = s_2^0 - s_1^0 - R \ln \frac{P_2}{P_1}$$

(420K, 290K)

$$\sigma = 2.04142 - 1.66802 - \frac{8.314}{28.97} \ln \frac{330}{100}$$

$$\sigma = 0.03076 \text{ kJ/kgK}$$

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vr Method

$$\frac{v_2}{v_1} = \left(\frac{RT_2}{p_2}\right)\left(\frac{p_1}{RT_1}\right)$$

$$pV = \frac{m}{M}\overline{R}T$$

$$pv = RT$$

$$\frac{P_2}{P_1} = \frac{P_{r(2s)}}{P_{r(1)}}$$

T_{2s}

$$\frac{v_2}{v_1} = \left[\frac{RT_2}{p_r(T_2)}\right]\left[\frac{p_r(T_1)}{RT_1}\right]$$

$$RT/p_r(T) \equiv v_r(T)$$

T_{2s}

$$\frac{v_2}{v_1} = \frac{v_{r2}}{v_{r1}} \qquad (s_1 = s_2, \text{ air only})$$

$$v_{r1} = v_r(T_1)$$

$$v_{r2} = v_r(T_{2s})$$

Tables A-22

In the end we need only one method, because it is the same, **pr** or **vr** = f (T), we will only use **pr** here.

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Other ideal gases

- IN THIS CASE WE DO NOT HAVE Pr TABLE, BUT WE CAN USE DE Cp METHOD.

The constant c_p method remains unchanged.

TABLE A-20

Ideal Gas Specific Heats of Some Common Gases (kJ/kg · K)

Temp. K	Air c_p	Air c_v	Air k	Nitrogen, N_2 c_p	Nitrogen, N_2 c_v	Nitrogen, N_2 k	Oxygen, O_2 c_p	Oxygen, O_2 c_v	Oxygen, O_2 k	Temp. K
250	1.003	0.716	1.401	1.039	0.742	1.400	0.913	0.653	1.398	250
300	1.005	0.718	1.400	1.039	0.743	1.400	0.918	0.658	1.395	300
350	1.008	0.721	1.398	1.041	0.744	1.399	0.928	0.668	1.389	350
400	1.013	0.726	1.395	1.044	0.747	1.397	0.941	0.681	1.382	400
450	1.020	0.733	1.391	1.049	0.752	1.395	0.956	0.696	1.373	450
500	1.029	0.742	1.387	1.056	0.759	1.391	0.972	0.712	1.365	500
550	1.040	0.753	1.381	1.065	0.768	1.387	0.988	0.728	1.358	550
600	1.051	0.764	1.376	1.075	0.778	1.382	1.003	0.743	1.350	600
650	1.063	0.776	1.370	1.086	0.789	1.376	1.017	0.758	1.343	650
700	1.075	0.788	1.364	1.098	0.801	1.371	1.031	0.771	1.337	700
750	1.087	0.800	1.359	1.110	0.813	1.365	1.043	0.783	1.332	750
800	1.099	0.812	1.354	1.121	0.825	1.360	1.054	0.794	1.327	800
900	1.121	0.834	1.344	1.145	0.849	1.349	1.074	0.814	1.319	900
1000	1.142	0.855	1.336	1.167	0.870	1.341	1.090	0.830	1.313	1000
Temp. K	**Carbon Dioxide, CO_2 c_p**	**Carbon Dioxide, CO_2 c_v**	**Carbon Dioxide, CO_2 k**	**Carbon Monoxide, CO c_p**	**Carbon Monoxide, CO c_v**	**Carbon Monoxide, CO k**	**Hydrogen, H_2 c_p**	**Hydrogen, H_2 c_v**	**Hydrogen, H_2 k**	**Temp. K**
250	0.791	0.602	1.314	1.039	0.743	1.400	14.051	9.927	1.416	250
300	0.846	0.657	1.288	1.040	0.744	1.399	14.307	10.183	1.405	300
350	0.895	0.706	1.268	1.043	0.746	1.398	14.427	10.302	1.400	350
400	0.939	0.750	1.252	1.047	0.751	1.395	14.476	10.352	1.398	400
450	0.978	0.790	1.239	1.054	0.757	1.392	14.501	10.377	1.398	450
500	1.014	0.825	1.229	1.063	0.767	1.387	14.513	10.389	1.397	500
550	1.046	0.857	1.220	1.075	0.778	1.382	14.530	10.405	1.396	550
600	1.075	0.886	1.213	1.087	0.790	1.376	14.546	10.422	1.396	600
650	1.102	0.913	1.207	1.100	0.803	1.370	14.571	10.447	1.395	650
700	1.126	0.937	1.202	1.113	0.816	1.364	14.604	10.480	1.394	700
750	1.148	0.959	1.197	1.126	0.829	1.358	14.645	10.521	1.392	750
800	1.169	0.980	1.193	1.139	0.842	1.353	14.695	10.570	1.390	800
900	1.204	1.015	1.186	1.163	0.866	1.343	14.822	10.698	1.385	900
1000	1.234	1.045	1.181	1.185	0.888	1.335	14.983	10.859	1.380	1000

Source: Adapted from K. Wark, *Thermodynamics*, 4th ed., McGraw-Hill, New York, 1983, as based on "Tables of Thermal Properties of Gases," NBS Circular 564, 1955.

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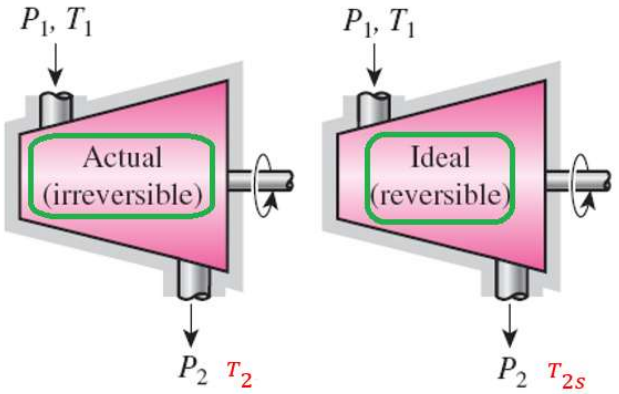
Method 2: Cp constant

$$s(T_2, p_2) - s(T_1, p_1) = c_p \ln\frac{T_2}{T_1} - R \ln\frac{p_2}{p_1} \qquad (6.22)$$



$$\eta_T = \frac{\dot{W}_a}{\dot{W}_s} = \frac{\dot{m}c_p(T_1 - T_2)}{\dot{m}c_p(T_1 - \boldsymbol{T_{2s}})} \qquad (1)$$

$$\frac{T_{2s}}{T_1} = \left(\frac{p_2}{p_1}\right)^{(k-1)/k} \qquad (2)$$

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- **AT LEAST YOU GENERATE YOUR OWN “Pr METHOD” !! Especially for gases of table A-23**

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- **AT LEAST YOU GENERATE YOUR OWN "Pr METHOD"**

For an ideal gas(other gas reported in kJ/mol.K) :

kJ/ kmol.K (universal)

per mole basis as

$$\bar{s}(T_2, p_2) - \bar{s}(T_1, p_1) = \bar{s}^\circ(T_2) - \bar{s}^\circ(T_1) - \bar{R} \ln \frac{p_2}{p_1} \qquad \text{(6.20b)}$$

Because s° depends only on temperature, it can be tabulated versus temperature, like h and u.

TABLE A-23

Table A-23

(Continued)

$\bar{h}$ and $\bar{u}$ in kJ/kmol. $\bar{s}^\circ$ in kJ/kmol · K

	Carbon Dioxide, CO_2 ($\bar{h}_f^\circ$ = −393,520 kJ/kmol)			Carbon Monoxide, CO ($\bar{h}_f^\circ$ = −110,530 kJ/kmol)			Water Vapor, H_2O ($\bar{h}_f^\circ$ = −241,820 kJ/kmol)			Oxygen, O_2 ($\bar{h}_f^\circ$ = 0 kJ/kmol)			Nitrogen, N_2 ($\bar{h}_f^\circ$ = 0 kJ/kmol)			
T(K)	$\bar{h}$	$\bar{u}$	$\bar{s}^\circ$	$\bar{h}$	$\bar{u}$	$\bar{s}^\circ$	$\bar{h}$	$\bar{u}$	$\bar{s}^\circ$	$\bar{h}$	$\bar{u}$	$\bar{s}^\circ$	$\bar{h}$	$\bar{u}$	$\bar{s}^\circ$	T(K)
600	22,280	17,291	243.199	17,611	12,622	218.204	20,402	15,413	212.920	17,929	12,940	226.346	17,563	12,574	212.066	600
610	22,754	17,683	243.983	17,915	12,843	218.708	20,765	15,693	213.529	18,250	13,178	226.877	17,864	12,792	212.564	610
620	23,231	18,076	244.758	18,221	13,066	219.205	21,130	15,975	214.122	18,572	13,417	227.400	18,166	13,011	213.055	620
630	23,709	18,471	245.524	18,527	13,289	219.695	21,495	16,257	214.707	18,895	13,657	227.918	18,468	13,230	213.541	630
640	24,190	18,869	246.282	18,833	13,512	220.179	21,862	16,541	215.285	19,219	13,898	228.429	18,772	13,450	214.018	640

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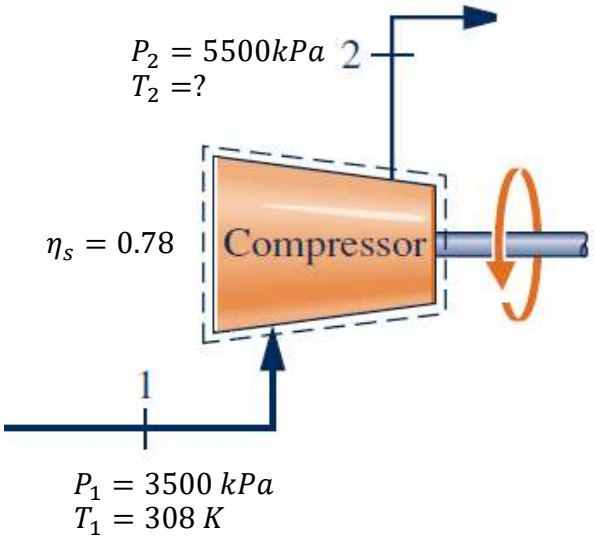
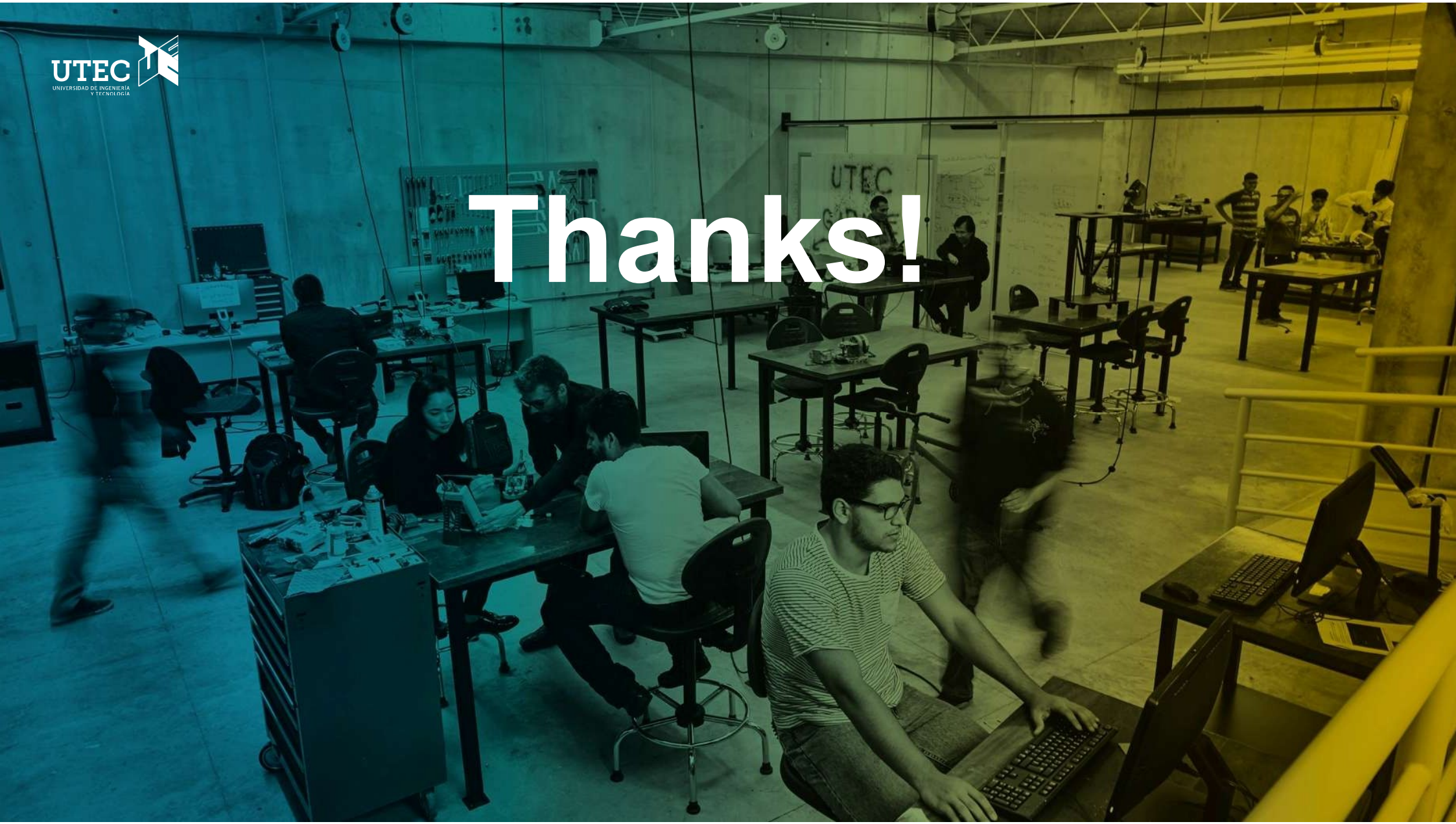
EXAMPLE 1.2 (OTHER GASES)

1.5 kg/s of methane is compressed in a pump station from 3500 kPa and 308 K to 5500 kPa. If the compressor efficiency is 0.78, how much power does the compressor need and what the methane temperature exit is? Methane can be considered as an ideal gas with Cp = 1.98 kJ/kg. K

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Thanks!